NHH

INSTITUTT FOR SAMFUNNSØKONOMI

DEPARTMENT OF ECONOMICS

SAM 18 2014

ISSN: 0804-6824

May 2014

Discussion paper

Energy Storage and Renewable Energy

BY

Tunç Durmaz

This series consists of papers with limited circulation, intended to stimulate discussion.

Norges Handelshøyskole

NORWEGIAN SCHOOL OF ECONOMICS

Energy Storage and Renewable Energy*

Tunç Durmaz†

15/5-2014

Abstract

I consider an economy with fossil fuel and renewable energy and energy storage, and search for the conditions that lead to welfare improvements when energy is stored. I then solve for the optimal decision rule and analyze the long-run tendencies of the economy-energy variables. The findings are threefold. First, energy storage is fostered by the convexity of the marginal utility (prudence), the marginal cost function for fossil fuel energy, and the degree of intermittency. Second, considering a low penetration of renewable energy to the power grid, energy storage is not welfare improving if the fossil fuel energy cost function is linear. Third, energy storage creates an added value to renewable energy investments when actively used. By showing the influence that energy storage can have on energy generation and investment decisions, I hope that the current work can be influential in a more generous treatment of energy supply in future energy-economy-climate models.

JEL codes: Q21, Q41, Q42, Q47, C61, C63, G31
Keywords: Energy storage; Fossil fuel energy; Renewable energy; Precautionary savings; Collocation method; Monte Carlo simulations

---

*I would like to thank François Salanie and Fred Schroyen for many constructive discussions and suggestions on the topic. I owe special thanks to Gunnar Eskeland for his encouragement and remarks. I have also benefited from the comments and suggestions of Burak Ünveren, Danilo Lopomo Beteto, Tiago Pinheiro and the participants of NHH ENE Brownbag Seminar and the Bergen Economics of Energy and Environment Research (BEEER) Conference. Lastly, I would like to thank the Center for Sustainable Energy Studies for research support.

†NHH, Norwegian School of Economics, Helleveien 30, 5045 Bergen, Norway. E-mail: Tunc.Durmaz@nhh.no

1 Introduction

The cost of renewable energy has been decreasing since the 1980s.[1] This may lead to optimism regarding the transformation of the energy industry. However, the growing concerns over man-made global warming show that the penetration of renewable energy to the power grid has only been gradual and insufficient to cover the increasing global energy demand.[2] As a result, fossil fuels still account for more than three quarters of global energy use and it is estimated that they will account for 78% by 2035 (EIA, 2011).

The real challenge may be found in the intermittent and variable nature of renewable energy that can cause difficulties in accessing energy when it is needed. If tomorrow's electric power grid is expected to contain a considerable amount of renewable energy, then the grid's stability, reliability and security may be at risk due to intermittency. In avoiding the exposure to such risks, energy storage technology (including (electrochemical) battery storage) will play a crucial role in the decades to come. Therefore, its modeling for long-term economic and policy analysis becomes an integral issue.[3]

Considering precautionary motives (prudence) and fossil fuel energy industry cost structures, the first aim of the current study is to construct an analytical framework and study the effects that an energy storage decisions can have on economic welfare and show how energy storage can contribute to the value of renewable energy investments when actively used. In view of the analytical results, my second aim is to show that the problem can be fully solved numerically in a long-term horizon and then calculate the long-run tendencies of the economic variables.

This study makes two primary contributions to the literature. To the best of my knowledge, it is the first to consider prudence in the energy economics literature. Second, considering the literature on precautionary savings, it is the first to analyze the effect of convex marginal cost function on savings decisions.

[1] As an example, the average PV module price (in constant 2005 prices) dropped from about 22\$/W in 1980 to less than 1.5\$/W in 2010. See Figure 3.17 in Arvizu et al. (2011).

[2] As is stated in IPCC (2013), it is with 95-100% probability that human influence has been the dominant source of the observed warming since the 1950s.

[3] Intermittency can be dealt with using renewable energy portfolios, for example portfolios of wind and solar farms. However, finding suitable land areas, convenient wind sites, and inadequate and costly transmission infrastructure are some of the difficulties.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the model and evaluates it under different scenarios. Calibration and simulation results are presented in Section 4. This is then followed by Section 5 where a discussion regarding the value of capacity increments in renewable energy is made. Numerical simulations for the value of capacity increments are presented in Section 6. Section 7 concludes. The description of the numerical method and a sensitivity analysis is presented in the Appendix.

2 Related Literature

The literature on energy storage to date has primarily focused on pumped hydroelectric storage.[4] Crampes and Moreaux (2001) develop an economic model that focuses on storage in the form of reservoirs for hydropower generation, which have a deterministic supply and compete with a thermal producer. The authors address the optimal energy mix and examine its compatibility with market mechanisms when the two producers compete. They show that optimal energy generated from the thermal station is determined by the industry specific costs and the intertemporal specification of utility.

In a two-period framework developed by the same authors, (Crampes and Moreaux, 2010) consider the optimal use of a pumped storage facility that consists of thermal and hydro energy technologies. In their model, hydro energy is generated from controlled inflows that require energy from the thermal technology. After solving for the optimal allocation, they show that there are social gains from storing water in an off-peak interval (where more energy from the thermal source is generated than consumed), which can then be used in the peak interval (where energy consumption will be more than energy generation).

Considering various cases such as fossil fuel or renewable energy generation with pumped hydroelectric storage, the economic fundamentals of the storage technology in a two-period model are examined by Forsund (2012). Given the growing interest in Norwegian hydroelectric reservoirs on the grounds that they will allow for a higher penetration of renewable energy into the European power

[4]An early paper on the management of an hydropower plant given uncertain inflows of water is that of Koopmans (1958).

grid, the paper also examines the effect of trade in electricity between regions. It finds that unless there are sufficiently large interconnection systems, the price differentials between the regions diminish. As a consequence, this reduces the scope for trade.

When there is a certain number of large conventional plants that have to be online (such as combined cycle gas turbines or the equivalent), intermittent wind energy and a planning horizon of 36 hours (hence one model period constitutes one hour), Tuohy and O'Malley (2011) show that, when modeling energy generation and dispatch of the power system, accounting for the intermittency is important in capturing the benefit of the flexibility offered by pumped storage. Accordingly, intermittent wind makes energy storage more attractive and its role becomes more significant when wind power is curtailed due to high wind.

The role of hydro storage in enabling a greater penetration of renewable energy into the grid has been investigated in Kanakasabapathy (2013), where the author looks at the impact of pumped storage energy trading on the sum of consumer and producer surplus of the individual market in a static model. The results show that while energy trading by pumped storage plants improve welfare in general, the economic implications for consumers and individual energy generators can be different.

In Korpaas et al. (2003) a method for the scheduling and operation of energy storage for wind power is presented.[5] Solving the optimization problem using dynamic programming, they show that energy storage enables wind power plant owners to take advantage of variations in the spot price, which in the end increases the value of wind power in electricity markets.

In a stylized model of energy investment and generation with two sources of energy, Ambec and Crampes (2012) address the optimal energy mix and analyze the optimal capacity investments in the absence of a storage technology. Hence, the focus is on the economics of the interplay between thermal and intermittent renewable energy and their capacities. After characterizing the optimal energy dispatch and capacities, they look at the economic policies that achieve first-best and second-best policies in decentralized markets.

In Van de Ven et al. (2011), the focus is on the decisions to satisfy the demand either directly from the grid or from the energy stored in batteries

[5]The analyzed duration of the model is 1 year, where each period constitutes 1 hour.

when the energy demand and prices are variable. Modeling the problem as a Markov decision process, they calculate a threshold to which the battery is charged whenever it is below the threshold, and discharged whenever it is above.

Our project, while sharing several characteristics of these papers, will depart from them in a significant way. In the presence of intermittency and balancing services, we investigate analytically the conditions that will cause welfare improvements when energy is stored, and show how prudence and the third-order derivative of the fossil fuel energy cost function can stimulate energy storage decisions. We also solve numerically for the optimal energy mix and storage decisions, i.e., the optimal decision rule, which we then supplement with Monte Carlo simulations in order to evaluate the long-run tendencies of the decision and state variables. Furthermore, we analyze how energy storage influences the value of renewable capacity increments and quantify this using numerical simulations.

3 The Model

Consider an infinite horizon economy with a representative consumer. There is a single-commodity, i.e., energy, which can be supplied from fossil fuels, renewables and energy storage systems:

$$Q_t = Q_{dt} + z_t Q_{ct} - R_t.$$

where Q_t is energy consumption, Q_{dt} is fossil fuel energy, Q_{ct} is the level for the renewable energy, $z_t \in [0,1]$ is current weather condition (normalized to one) that is known prior to taking economic decisions, and R_t represents the energy storage decision.[6] When R is positive, energy is stored in order to be used in the following periods, and when it is negative the stored energy is used.

The equation of motion for the stored energy is the following:

$$S_{t+1} = \phi S_t + R_t,$$

where S_t is the level of stored energy at time t. Whenever energy is stored, a certain percentage of it will be lost in time. This is captured by the round-trip

[6] We abstract from exhaustibility of fossil fuels.

efficiency parameter, $\phi \in (0, 1)$, which is the ratio of the energy recovered to the initially stored energy.

The timing of the model is depicted in Figure 1. At the beginning of period t, the economy inherits stored energy, S_t. Having observed S_t and the weather conditions, z_t, the fossil fuel and renewable energy decisions, Q_{dt} and Q_{ct}, are made. After taking into account the loss in stored energy, $(1-\phi)S_t$, and Q_{dt} and Q_{ct}, the levels for energy storage, R_t, and, therefore, energy consumption, Q_t, are decided. We assume that the production and consumption almost coincide so that no energy is lost in this process. Given the energy storage decision, R_t, the level of stored energy transferred into period t+1 is $S_{t+1} = \phi S_t + R_t$.

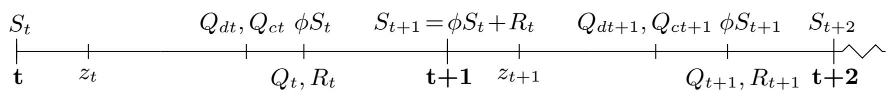


Figure 1: *Timing of the model*

We assume that energy demand is stationary (Førsund, 2007, Ch. 9). $U(Q_t)$ is the per period utility function, which satisfies the standard monotonicity and concavity assumptions. Preferences over energy consumption take the additively separable form given by:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t U(Q_t)\right], \tag{1}$$

where $0 < \delta < 1$ is the discount factor and $\mathbb{E}(\cdot)$ denotes the expected value with respect to the probability distribution of the random variables $\{Q_t\}_{t=1}^{\infty}$.

The unit cost of fossil fuel energy is either constant, $C_d'(Q_d) > 0, C_d''(Q_d) = 0$, increasing, $C_d'(Q_d) > 0, C_d''(Q_d) > 0, C_d'''(Q_d) = 0$, or increasingly-increasing, $C_d'(Q_d) > 0, C_d''(Q_d) > 0, C_d'''(Q_d) > 0$ where $C_d'(Q_d)$, $C_d''(Q_d)$ and $C_d'''(Q_d)$ are the first, second and third-order derivatives, respectively, of the cost function with respect to Q_d.[7] When the unit cost is constant, one can relate this to a *constant-cost industry*, in which the input price (price of fossil fuels), is constant.[8] On the other hand, when the cost function is convex, this resembles

[7] We do not consider concave cost functions.

[8] As an example, think of a fossil fuel power plant that does not have market power and therefore takes the price as given (assume that the industry has a long-term agreement regarding the price of the inputs, which makes it secure against changes in fossil fuel prices).

an *increasing-cost industry*.[9] Moreover, when the third-order derivative of the cost function is strictly positive, $C_d'''(Q_d) > 0$, we will call this an *increasingly increasing cost industry*. Lastly, $C_c(Q_c)$ is the cost function for the renewable energy generation. As the cost structure for the renewable energy will be discussed later on, we do not make any assumption regarding its functional form at the moment.

When solving the energy generation problem, the aim is to maximize (1), the intertemporal welfare of the representative agent, through energy generation and energy storage. For S_0 being the inherited energy and z_0 the initial weather condition, the planner's problem, formulated in the form of a Bellman equation, is the following:

$$\begin{aligned} V(S_t, z_t) = & \max_{\{Q_t, Q_{dt}, Q_{ct}, R_t, S_{t+1}\}} \big\{ U(Q_t) - C_d(Q_{dt}) - C_c(Q_{ct}) + \delta \mathbb{E}_z \left[V(S_{t+1}, z_{t+1}) \right] \big\} \\ \text{s.t} \;\; & Q_t = Q_{dt} + z_t Q_{ct} - R_t, \\ & S_{t+1} = \phi S_t + R_t, \\ & \bar{Q}_d \geq Q_{dt} \geq \underline{Q}_d, \\ & \bar{Q}_c \geq Q_{ct} \geq 0, \\ & \bar{S} \geq S_t \geq 0, \\ & S_0 \geq 0, \; 1 \geq z_0 \geq 0, \end{aligned} \tag{2}$$

where $V(S_t, z_t)$ is the value function, which is the maximum attainable sum of the current and future rewards given the current (inherited) level for stored energy, S_t and current weather conditions, z_t. Having observed z_t, the economy produces Q_{dt} and Q_{ct}, and decides whether to store or use the stored energy R_t. When Q_{dt}, Q_{ct} and R_t are chosen, the energy consumption becomes $Q_t = Q_{dt} + z_t Q_{ct} - R_t$.

Future weather conditions, $z' \equiv z_{t+1} \in [0, 1]$ are imperfectly known *ex ante* and the surrounding uncertainty is removed only at the end of the current period –after Q_{dt}, Q_{ct} and R_t are determined. $\mathbb{E}_z$ denotes the expectation operator over the distribution for z', which satisfies the i.i.d property, is described by a density function $f(z)$, and the distribution function corresponding to $f(z)$ is denoted by

[9] Assume that there is a unique merit order of using individual generators, so that first the power plants with the lower marginal costs of energy generation would be brought on line (like a coal-fired power plant), followed by costlier ones (such as a natural gas power plant with carbon capture and storage).

$F(z) : z' \sim F(z) = \Pr(z' \leq z)$. z' being stochastic makes the renewable energy generation random. In other words, even though energy can be generated from wind turbines when there is enough wind or from solar panels when the sun is brightly shining, these sources are beyond our control and can only be forecasted with some degree of accuracy, hence with some degree of uncertainty.

$\bar{Q}_d$, $\bar{Q}_c$ and $\bar{S}$ are the capacity constraints for fossil fuel energy, renewable energy, and energy storage, respectively. We assume a big enough capacity for fossil fuel energy throughout the analysis such that it never binds. $\underline{Q}_d$ is the minimum level of fossil fuel energy generation to avoid ramp-up problems.[10]

We focus only on the cases in which renewable energy operates at its capacity at all times, $z_t\bar{Q}_c$ for $t = 0, 1, 2, ..., \infty$: once there is an installed capacity for renewable energy, the unit cost of renewable energy becomes so low that we can take it as zero (Ambec and Crampes, 2012; Førsund and Hjalmarsson, 2011). Hence, for $C_c'(Q_c) = 0$, the only cost in generating renewable energy is the opportunity cost of not generating more energy than $\bar{Q}_c$.

Given that $F(z)$ and the model parameters are time invariant, the problem is stationary, i.e., the problem faced by the planner at every period is identical: $V_t(S, z) = V_{t+y}(S, z)$ for all $y > 0$. Therefore, we shall drop the time subscripts and use primes to denote next-period values (not to confuse with partial derivatives). Then the dynamic stochastic decision problem has the following structure. At every period, the planner observes the state of the economy, i.e. how much energy storage has been inherited and the state of the weather conditions, say how strong the wind blows and the sun shines, denoted by (S, z), and decides on the optimal actions (Q, Q_d, R, S'). Therefore, the planner searches for an optimal decision rule $\{Q^*(S, z), Q_d^*(S, z), R^*(S, z), S'^*(S, z)\}$ that solves $V(S, z)$.[11]

Our problem is not fully tractable analytically. Therefore, we leave the problem of finding the optimal decision rule to the numerical section. However, this does not preclude us from analyzing the welfare effects of engaging in energy storage in the absence of stored energy. We will therefore assume that there is no inherited energy, $S = 0$, and energy is not stored, $R = 0$, hence $S' = 0$, and

[10] Once shut down, it can take a long time to ramp-up a fossil fuel power plant, which may then lead to a power shortage.

[11] As we assume no externalities, it is straightforward to implement the socially optimal allocation in a decentralized equilibrium.

then ask whether or not a marginal increase in S' is welfare improving.

The Bellman equation is the following:

$$V(S,z) = \max_{Q_d} \left\{ U(Q_d + z\bar{Q}_c - S' + \phi S) - C_d(Q_d) + \delta \mathbb{E}_{z'}\left[V(S',z')\right] \right\}\Big|_{S'=0}$$
$$\text{s.t} \quad Q_d \geq \underline{Q}_d, \tag{3}$$

for which FOC wrt Q_d gives:

$$U'(Q) - C_d'(Q_d) \leq 0, \tag{4a}$$
$$U''(Q) - C_d''(Q_d) < 0, \tag{4b}$$

where the second expression gives the second-order condition for a maximum.

Let the optimal decision (the optimal response function) be $Q_d(S,z)$. Then for κ ($\kappa = z, z'$) we have:

$$\begin{array}{lll} Q_d(S,\kappa) = \underline{Q}_d, & \text{if} & U'(\underline{Q}_d + \kappa\bar{Q}_c + \phi S) \leq C_d'(\underline{Q}_d), \\ Q_d(S,\kappa) > \underline{Q}_d, & \text{otherwise} & U'(\underline{Q}_d + \kappa\bar{Q}_c + \phi S) > C_d'(\underline{Q}_d), \end{array} \tag{5}$$

where in the first conditional statement, the marginal cost of generating the fossil fuel energy is bigger than the marginal utility coming from its consumption when $Q_d = \underline{Q}_d$. Hence, there is a corner solution for the fossil fuel energy. In the second conditional statement, the solution is interior. Therefore, $U'(Q(S,\kappa)) = C_d(Q_d(S,\kappa))$.

The first conditional statement implies that there is a threshold level $\bar{z}(\bar{Q}_c)$ (slightly abusing notation, we write $\bar{z}(\bar{Q}_c, \underline{Q}_d, S)$ as $\bar{z}(\bar{Q}_c)$),

$$\bar{z}(\bar{Q}_c) \equiv \frac{U'^{-1}(C_d'(\underline{Q}_d)) - \underline{Q}_d - \phi S}{\bar{Q}_c}, \tag{6}$$

such that for $z > \bar{z}(\bar{Q}_c)$, the renewable energy generation, $z\bar{Q}_c$, takes high enough values so that Q_d has a corner solution. On the other hand, when $z < \bar{z}(\bar{Q}_c)$, i.e., when the renewable energy generated is low, then $Q_d(S,z) > \underline{Q}_d$.

Now let us ask what will be the welfare effect if S' is increased marginally from zero:

$$\left.\frac{\partial\left\{V(S,z)\right\}}{\partial S'}\right|_{S'=0} = \left.\frac{\partial\left\{U(Q_d + z\bar{Q}_c - S' + \phi S) - C_d(Q_d) + \delta\mathbb{E}_{z'}\left[V(S',z')\right]\right\}}{\partial S'}\right|_{S'=0},$$

which gives:

$$\left.\frac{\partial\left\{\cdot\right\}}{\partial S'}\right|_{S'=0} = -U'(Q_d(S,z) + z\bar{Q}_c + \phi S) + \delta\ \mathbb{E}_{z'}\left[\frac{\partial V(S',z')}{\partial S'}\right]\Bigg|_{S'=0} \qquad (7)$$

From the Envelope Theorem, only the direct effect of a marginal change in the state variable matters on the value function. Given that we evaluate the problem when $S' = 0$, the derivative of the associated value function w.r.t S shows:

$$V_1(S,z) = \phi U'(Q_d(S,z) + z\bar{Q}_c + \phi S),$$

where $V_1(\cdot)$ is the derivative of the value function with respect to its first argument. This is the Benveniste-Scheinkman (Envelope Theorem) condition. Iterating this one period forward gives:

$$V_1(S',z') = \phi U'(Q_d(S',z') + z'\bar{Q}_c + \phi S').$$

By plugging this result in (7), one gets:

$$\left.\frac{\partial\left\{\cdot\right\}}{\partial S'}\right|_{S'=0} = -U'(Q_d(S,z) + z\bar{Q}_c + \phi S) + \delta\phi\mathbb{E}_{z'}\left[U'(Q_d(0,z') + z'\bar{Q}_c)\right]. \quad (8)$$

As we restrict the analysis to $S = 0$, and hence assume no inherited energy, then from (8) we have:

$$\left.\frac{\partial\left\{\cdot\right\}}{\partial S'}\right|_{S'=0} = -U'(Q_d(0,z) + z\bar{Q}_c) + \delta\phi\mathbb{E}_{z'}\left[U'(Q_d(0,z') + z'\bar{Q}_c)\right]. \qquad (9)$$

By decomposing the term into realizations of z', such that there is a corner solution for Q'_d, i.e., $z > \bar{z}(\bar{Q}_c)$, and realizations for which there is an interior solution, so that $U'(Q_d(0,z') + z'\bar{Q}_c) = C'_d(Q_d(0,z'))$ (see equations (5) and

(6)), $\mathbb{E}_{z'}\left[U'(Q_d(0,z')+z'\bar{Q}_c)\right]$ can then be presented as:

$$\begin{aligned}\mathbb{E}_{z'}\left[U'(Q_d(0,z')+z'\bar{Q}_c)\right] =& \mathbb{E}_{z'}\left[C_d'(Q_d(0,z'))|z'<\bar{z})\right]\Pr(z'<\bar{z}) \\ &+\mathbb{E}_{z'}\left[U'(\underline{Q}_d+z'\bar{Q}_c|z'>\bar{z})\right]\Pr(z'>\bar{z}).\end{aligned} \tag{10}$$

Let $g(z') \stackrel{\text{def}}{=} C_d'(Q_d(0,z'))$ and $h(z') \stackrel{\text{def}}{=} U'(\underline{Q}_d+z'\bar{Q}_c)$. Taking the expectation of a second-order Taylor approximation around $\check{z} \equiv \mathbb{E}[z'|z'<\bar{z}]$ for the former and $\hat{z} \equiv \mathbb{E}[z|z'>\bar{z}]$ for the latter gives:

$$\begin{aligned}\mathbb{E}[g(z')] &\simeq g(\check{z})+\frac{1}{2}g''(\check{z})\sigma^2_{\check{z}}, \\ \mathbb{E}[h(z')] &\simeq h(\hat{z})+\frac{1}{2}h''(\hat{z})\sigma^2_{\hat{z}},\end{aligned} \tag{11}$$

where $\sigma^2_{\check{z}}$ and $\sigma^2_{\hat{z}}$ are the conditional variances of the random variable z' given $z'<\bar{z}$ and $z'>\bar{z}$, respectively.

We are interested in calculating $g''(\check{z})$ and $h''(\hat{z})$. Firstly, $g'(z') = C_d''(Q_d(0,z'))\frac{\partial Q_d(0,z')}{\partial z'}$, where,

$$\frac{\partial Q_d(0,z')}{\partial z'} = \frac{U''(Q(0,z'))\bar{Q}_c}{C_d''(Q_d(0,z'))-U''(Q(0,z'))} < 0, \tag{12}$$

Following (12) one gets,

$$g''(z') = C_d'''\left(\frac{\partial Q_d(0,z')}{\partial z'}\right)^2 + C_d''\frac{\partial^2 Q_d(0,z')}{\partial z'^2}, \tag{13}$$

and,

$$\frac{\partial^2 Q_d(0,z')}{\partial z'^2} = \frac{\bar{Q}_c^2}{(C_d''-U'')^3}\left(C_d''^2U'''-U''^2C_d'''\right),$$

where $U'''(\cdot)$ is the third-order derivative of the utility function.

Using these results, one then arrives at the following expression for $g''(\hat{z})$:

$$g''(z') = \bar{Q}_c^2\left[\frac{(C_d'')^3}{(C_d''-U'')^3}U''' + \frac{(-U'')^3}{(C_d''-U'')^3}C_d'''\right]. \tag{14}$$

where the term in the square brackets is a weighted average of $U'''(\cdot)$ and $C_d'''(\cdot)$.

Lastly,

$$h'(z') = \bar{Q}_c U''(\underline{Q}_d + z'\bar{Q}_c), \tag{15a}$$

$$h''(\hat{z}) = \bar{Q}_c^2 U'''(\underline{Q}_d + \hat{z}\bar{Q}_c). \tag{15b}$$

Using the results from the second-order Taylor approximation, (11), (14) and (15b), (10) becomes the following:

$$\begin{aligned} \mathbb{E}_{z'}\left[U'(Q_d(0,z') + z'\bar{Q}_c)\right] = &\left(C_d'(Q_d(0,\check{z})) + \frac{1}{2}g''(\check{z})\sigma_{\check{z}}^2\right)\Pr(z' \le \bar{z}) \\ &+ \left(U'(\underline{Q}_d + \hat{z}\bar{Q}_c) + \frac{1}{2}h''(\hat{z})\sigma_{\hat{z}}^2\right)\Pr(z' > \bar{z}). \end{aligned} \tag{16}$$

From (9),(14),(15b) and (16), the welfare effect of increasing S' marginally from zero when $S = 0$ can then be shown as:

$$\begin{aligned} \left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} &= -U'(Q_d(0,z) + z\bar{Q}_c) \\ &+ \delta\phi\Bigg[\left(C_d'(Q_d(0,\check{z})) + \frac{1}{2}\bar{Q}_c^2\left[\frac{(C_d'')^3}{(C_d'' - U'')^3}U''' + \frac{(-U'')^3}{(C_d'' - U'')^3}C_d'''\right]\sigma_{\check{z}}^2\right)\Pr(z' \le \bar{z}) \\ &+ \left(U'(\underline{Q}_d + \hat{z}\bar{Q}_c) + \frac{1}{2}\bar{Q}_c^2 U'''(\underline{Q}_d + \hat{z}\bar{Q}_c)\sigma_{\hat{z}}^2\right)\Pr(z' > \bar{z})\Bigg]. \end{aligned} \tag{17}$$

Following equation (17), we can establish the following:

Proposition 3.1. If the cost of engaging in energy storage is sufficiently low and the benefit expected from storing energy is sufficiently high, energy storage is welfare improving. Convexity in the marginal utility, i.e., prudence, and in the marginal cost function in fossil fuel energy generation, and the degree of intermittency are factors that foster energy storage decisions.

Notice from the expression given by (17), the value on the RHS diminishes in the absence of prudence. Therefore, the convexity of marginal utility is a crucial factor that increases the willingness of the economy to engage in energy storage.

One other thing we can notice from expression (17) is that a convex marginal cost of fossil fuel energy does play a significant role in determining the impact of uncertainty on the optimal energy storage strategy. Surprisingly, it can be seen that even in the absence of prudence, a non-negative C_d''' alone is necessary for "*precautionary*" saving of energy.

Notice that the decision to engage in energy storage given the intermittency (uncertainty) in renewable energy relates to the literature on precautionary saving, where a positive third-order derivative of the utility function governs the precautionary behavior. The analysis regarding the precautionary saving under uncertainty was first introduced by Leland (1968) and Sandmo (1970). A modern treatment of precautionary saving can be found in Kimball (1990), where he coins the term 'prudence' when the marginal utility of consumption is convex, and shows that prudence is sufficient for a demand in precautionary savings in standard intertemporal models of consumption.

Regarding the convexity of the marginal cost, one can imagine an implicitly assigned capacity constraint –an upper bound– on the fossil fuel energy, which will induce the effect from convexity to become predominant when fossil fuel energy is required to take high generation levels. As a consequence, for a limited fossil fuel energy capacity, such an effect can be quite fundamental.

As a special case, assume that $z = z' = 0$. This is to say that the renewable energy either does not exist or is completely inefficient. This will naturally cause fossil fuel energy generation to be over its ramp-up level, $Q_d > \underline{Q}_d$, and $\mathbb{E}[z] = \check{z} = \hat{z} = \sigma_z^2 = \sigma_{\check{z}}^2 = \sigma_{\hat{z}}^2 = 0$, where σ_z^2 is the variance of the probability distribution for z'. Moreover, $\Pr(z' \leq \bar{z}) = 1$, while $\Pr(z' > \bar{z}) = 0$. As a result, the latter term in (9) becomes:

$$\delta\phi\mathbb{E}_{z'}\left[U'(Q_d(0,z') + z'\bar{Q}_c)\right] = \delta\phi C_d'(Q_d(0,0)).$$

Using (16), we then have the following welfare effect when S' is increased marginally from zero:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)C_d'(Q_d(0,0)) < 0. \tag{18}$$

From (18), we can establish the following corollary:

Corollary 3.2. If an economy does not have access to renewable energy, then storing energy is welfare deteriorating.

The intuition is that as the resource used for storing energy comes from fossil fuel energy generation, the marginal resource, then the unit cost of storing energy is $C_d'(Q_d(0,0))$. When energy is stored, its present value adjusted for the discount factor and the loss in energy becomes $\delta\phi C_d'(Q_d(0,0))$. Comparing the cost of storing energy to its value adjusted for the discount factor and the round-trip efficiency, it is seen from (18) that energy storage is suboptimal: storing energy in the ground (or in the fuel itself) is more efficient. As a result, energy consumption, Q, equals fossil fuel energy generation, Q_d, in every period.

Suppose now that the renewable energy exists and is efficient. As another special case, let there be no intermittency problem and let z take the same level at every period: $z = z' = \mathbb{E}[z] = \mathbb{E}[z']$. Therefore, $\sigma_z^2 = \sigma_{\check{z}}^2 = \sigma_{\hat{z}}^2 = 0$. If $z < \check{z}$, then $Q_d(0,z) > \underline{Q}_d$ and $U'(Q_d(0,z) + z\bar{Q}_c) = C_d'(Q_d(0,z))$ at all times. Following this, from (17) one arrives at the following:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} \simeq -(1-\delta\phi)C_d'(Q_d(0,z)) < 0. \tag{19}$$

Conversely, if $z > \check{z}$, then $U'(\underline{Q}_d + z\bar{Q}_c) \leq C_d'(\underline{Q}_d)$ and $Q_d(0,z) = \underline{Q}_d$ always. As a result, from (17), one gets:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} \simeq -(1-\delta\phi)U'(\underline{Q}_d + z\bar{Q}_c) < 0. \tag{20}$$

Following (19) and (20), we can establish the following corollary:

Corollary 3.3. In an economy with fossil fuel and renewable energy, storing energy is welfare deteriorating in the absence of the intermittency problem.

Intermittency in renewable energy, hence uncertainty in the levels of energy generated by the renewable energy capacity, is the cause that assigns a positive value to energy storage. Without it, it will only be welfare deteriorating to engage in energy storage.

Suppose now that the renewable energy is intermittent (back to the reality). In such a setting, one can come across a setup in which $z < \bar{z}$ always. This is to say that, the penetration of renewable energy into the power grid is low and after deducting the ramp-up level for fossil fuel energy generation, the renewable energy generation can never be enough to satisfy the remaining energy demand for the economy: $z\bar{Q}_c < \bar{z}\bar{Q}_c = U'^{-1}(C_d'(\underline{Q}_d) - \underline{Q}_d)$. This results in $\Pr(z' > \bar{z}) = 0$, and therefore, from (16), $\mathbb{E}_{z'}\left[U'(Q_d(0,z') + z'\bar{Q}_c)\right] = \left(C_d'(Q_d(0,\check{z})) + \frac{1}{2}g''(\check{z})\sigma_{\check{z}}^2\right)$. Given that there is an interior solution for Q_d in the economy, hence $U'(Q_d(0,z) + z\bar{Q}_c) = C_d'(Q_d(0,z))$, one can write (17) as:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -C_d'(Q_d(0,z)) + \delta\phi\left(C_d'(Q_d(0,\check{z})) + \frac{1}{2}g''(\check{z})\sigma_{\check{z}}^2\right) \tag{21}$$

As a starting point, suppose that $z = \check{z}$. We then have:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)C_d'(Q_d(0,z)) + \frac{1}{2}\delta\phi g''(\check{z})\sigma_{\check{z}}^2 \tag{22}$$

It can be seen that $U''' > 0$ or $C_d''' > 0$ is a necessary condition for engaging in energy storage. Therefore, convexity in the marginal utility (prudence) and in the marginal cost function play a major role in energy storage decisions. Additionally, if $z > \check{z}$, then $Q_d(0,z) < Q_d(0,\check{z})$, and it becomes more likely to engage in storing energy. Conversely, if $z < \check{z}$, then $Q_d(0,z) > Q_d(0,\check{z})$, and storing energy may be welfare deteriorating.

If one assumes a linear cost function, from (14), one gets $g''(\check{z}) = 0$. Then (22) becomes:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)c_d < 0, \tag{23}$$

where $c_d = C_d'(Q_d(0,z))$, which is a positive constant, is the marginal cost of generating fossil fuel energy when the cost function is linear.

Corollary 3.4. If the renewable energy capacity, $\bar{Q}_c$, is small so that the fossil fuel energy generation is always above its ramp-up level, $Q_d > \underline{Q}_d$, and the fossil fuel energy cost function is linear, i.e., there is a constant-cost fossil fuel energy industry, then storing energy is welfare deteriorating and therefore is never optimal. Prudence, the positive third-order derivative of the utility function, loses its impact on storage decisions.

The result follows from (23).

The intuition is that in an economy in which the penetration of the renewable energy to the power grid is low, the dirty carrier generates energy over the ramp-up level and becomes the source for energy storage. This naturally means that the present value adjusted for the discount factor and the loss in energy becomes $\delta\phi c_d$, which is smaller than c_d. It can then be seen from (23) that energy storage turns out suboptimal. Hence, although the renewable energy is stochastic, there is indeed no real risk in the economy as long as the dirty carrier has no barriers to produce energy in the following period. Therefore, storage technology will not be employed even if it is perfectly efficient.

Now assume that the renewable energy capacity is considerably high and there is a favorable distribution for z such that $Q_d = \underline{Q}_d$ at all times. Here, we depart from the previous assumption, $z \in [0,1]$, and assume $z \in (\bar{z}, 1]$, as if the wind never subsides and is always above $\bar{z}$. Considering the welfare effects of a marginal increase in S' we have:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -U'(\underline{Q}_d + z\bar{Q}_c) + \delta\phi\mathbb{E}_{z'}\left[U'(\underline{Q}_d + z'\bar{Q}_c)\right]\Big|_{S'=0}. \tag{24}$$

Given that $\Pr(z' \leq \bar{z}) = 0$ (and $\Pr(z' > \bar{z}) = 1$), one gets the following welfare effect from a marginal increase in S' using (17):

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -U'(\underline{Q}_d + z\bar{Q}_c) + \delta\phi U'(\underline{Q}_d + \hat{z}\bar{Q}_c) + \frac{1}{2}\delta\phi\bar{Q}_c^2 U'''(\underline{Q}_d + \hat{z}\bar{Q}_c)\sigma_z^2. \tag{25}$$

To fix ideas, suppose that the current realization of z coincides with its expected future realization, i.e., $z = \hat{z}$. Then:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)U'(\underline{Q}_d + \hat{z}\bar{Q}_c) + \frac{1}{2}\delta\phi\bar{Q}_c^2 U'''(\underline{Q}_d + \hat{z}\bar{Q}_c)\sigma_z^2. \tag{26}$$

One sees that U''' is a necessary condition for storage to be optimal in this case. Due to the concavity of the utility function, if $z > \hat{z}$, then $U'(\underline{Q}_d + z\bar{Q}_c) < U'(\underline{Q}_d + \hat{z}\bar{Q}_c)$ and it becomes more likely that the economy will engage in energy storage. Conversely, if $z < \hat{z}$, then $U'(\underline{Q}_d + z\bar{Q}_c) > U'(\underline{Q}_d + \hat{z}\bar{Q}_c)$, and it becomes less likely to start storing energy.

4 Numerical Analysis

In solving the dynamic stochastic decision problem given by (2), we employ dynamic programming based on Bellman's principle of optimality: regardless of the decisions taken to enter a particular state in a particular stage, any optimal policy has the property that the remaining decisions given the stage resulting from the current decision must constitute an optimal policy. Hence, we look for an optimal decision rule $\{Q^*(S,z), Q_d^*(S,z), Q_c^*(S,z), R^*(S,z), S'^*(S,z)\}$, which solves $V(S,z)$.

In order to make sure that the numerical problem has a solution and this solution is unique, we establish the contraction property of the dynamic program. The right hand side of the Bellman equation is a mapping of the value function $V(\cdot)$ and $V = TV$ is a fixed point of the mapping, where T is a function mapping V into itself. For there to be a unique solution to the dynamic programming problem, we need show that the mapping for the Bellman equation above is indeed a contraction mapping. In showing that a mapping is a contraction, we make use of Blackwell's sufficient conditions for a contraction (see Appendix A).

Proposition 4.1. The energy generation and storage model we work with satisfies Blackwell's sufficient conditions for a contraction. Therefore there exists a unique fixed point for the mapping of the value function, i.e., a unique solution to the dynamic programming problem.

Proof. See Appendix B.

4.1 Calibration

Our purpose with the simulations is not to provide a comprehensive quantitative evaluation. Rather, we want to highlight the roles different industry cost structures and precautionary motives can play in an economy equipped with fossil fuel and renewable energy, and energy storage capacities.

Suppose there exists an economy in which the level of energy consumption is Q = 450MW/h (megawatts per hour), which is supplied by a fossil fueled power plant initially.[12] Then the fossil fuel energy generation, Q_d equals energy consumption, Q: $Q = Q_d = 450$MW/h.

In the economy the energy demand is assumed to be stable. As hourly energy generation data can bias the analysis, we focus on weekly data: $Q = Q_d = 450$ MW/h $\times$ 24h/d $\times$ 7d/w $=$ 75600 MW/w $=$ 75.6GW/w, where h,d,w stand for hour, day, week, respectively.[13] For ease of notation, we drop 'per time period' notation and focus only on the thermal unit, GW. We take an annual discount rate of 5%. This corresponds to a weekly discount factor, $\delta = 0.9991$.

For the fossil fuel power plant, we assume that the ramp-up level equals $\underline{Q}_d = 8.4$GW, corresponding to 50MW per hour. The capacity constraint for fossil fuel power generation is given by $\bar{Q}_d = 100.8$GW corresponding to 600MW, which, in the simulations, will not bind as $Q = 75.6$GW.

In the simulations, we will make use of a constant relative risk aversion (CRRA) utility function, $Q^{1-\gamma}/(1-\gamma)$, where γ and $\gamma + 1$ are the coefficients of relative risk aversion and relative prudence, respectively. We take $\gamma = 2$.[14] From the necessary first-order condition w.r.t Q_d, given by (4a), we then have $Q^{-\gamma} = C_d'(Q_d)$. Assuming a linear cost function for fossil fuel energy, $C_d(Q_d) = c_l Q_d$, where c_l is a constant, one then gets, $c_l = Q^{-\gamma}$. For $Q = 75.6$GW, $c_l = 0.000175$UoN (units of the numeraire). If, however, the cost function is quadratic, we have $C_d(Q_d) = c_q Q_d^2$, where c_q is another constant. Finally, for a cubic cost function we have $C_d(Q_d) = c_c Q_d^3$, where c_c is also a constant.

In order to be consistent in the analysis, we assume that when the fossil fuel energy generation is at the ramp-up level, $Q_d = \underline{Q}_d$, the marginal costs are equal among the different cost functions. This then gives us:

$$c_l = 2c_q \underline{Q}_d = 3c_c \underline{Q}_d^2 \tag{27}$$

[12] Although we do not aim for a comprehensive quantitative evaluation, it is still possible to find a range of examples to associate with 450MW/h of energy consumption. As an example, electricity peak demand in Uganda is around 450MW/h (EIU, 2013). Also, an island in Greece, Agathonisi, has an annual electricity consumption of 450 MW/h (Kaldellis et al., 2012).

[13] 1GW (gigawatt) = 1000 MW.

[14] (Heal, 2009) argues that $\gamma \in [2, 6]$ would be a reasonable range.

Hence, using the result that $c_l = 0.000175$, we get:

$$c_q = \frac{c_l}{2\underline{Q}_d} = 1.0417 \text{ x } 10^{-5} \quad (28)$$

$$c_c = \frac{c_l}{3\underline{Q}_d^2} = 8.2672 \text{ x } 10^{-7}. \quad (29)$$

For $Q_d > \underline{Q}_d$, we then have $c_l < 2c_q Q_d < 3c_c Q_d^2$.

Suppose that a wind farm with a maximum capacity of $\bar{Q}_c = 100.8$GW, which corresponds to 600MW per hour is then introduced to the economy.[15] Moreover, the economy gains access to energy storage technology with a maximum capacity of 100MW, which corresponds to $\bar{S} = 16.8$GW per week.[16] We first assume that 1% of stored energy would be lost every week, hence $\phi = 0.99$. We address the effects of different round-trip efficiency parameters by making a sensitivity analysis in Appendix E.1.

As is discussed in the Appendix for method description (Appendix C.1), we approximate the expected value for the intermittent renewable energy production, Q_c, using Gaussian quadrature nodes and weights. In determining the weights and nodes (normalized wind speed), we make use of a beta distribution defined on the interval [0, 1] and parametrized by two positive shape parameters, a and b. As an example, for $a = 2$ and $b = 2$, the probability density function, $f(z)$, for the beta distribution looks like the one in Figure 2.

Finally, in evaluating the long-run steady state behavior of the controlled economic process, we will make use of Monte Carlo Simulations (see Appendix C.1).

[15] The Fantanele-Cogealac Wind Farm, which opened in 2012 in Romania, and the Whitelee Wind Farm, which opened in 2012 in the United Kingdom, have capacities of 108GW and 90.5GW, respectively.

[16] Considering battery storage, even though such a capacity is not present as of today, it is achievable given the current battery technology. The biggest battery storage capacity exists in west Texas located at 153 MW Notrees wind farm where 36 MW battery storage system became operational in December 2012. The 36 MW battery storage is a scalable assembly of thousands of 12 volt, 1 kWh, dry cell batteries based on a proprietary formula of alloys including copper, lead and tellurium.

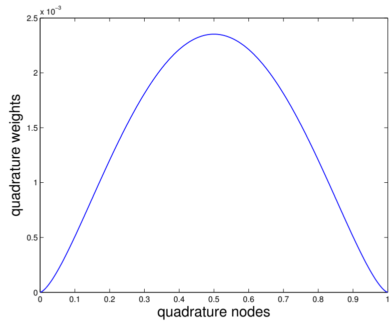


Figure 2: *Beta probability density function for the (normalized) wind speed (a=2, b=2).*

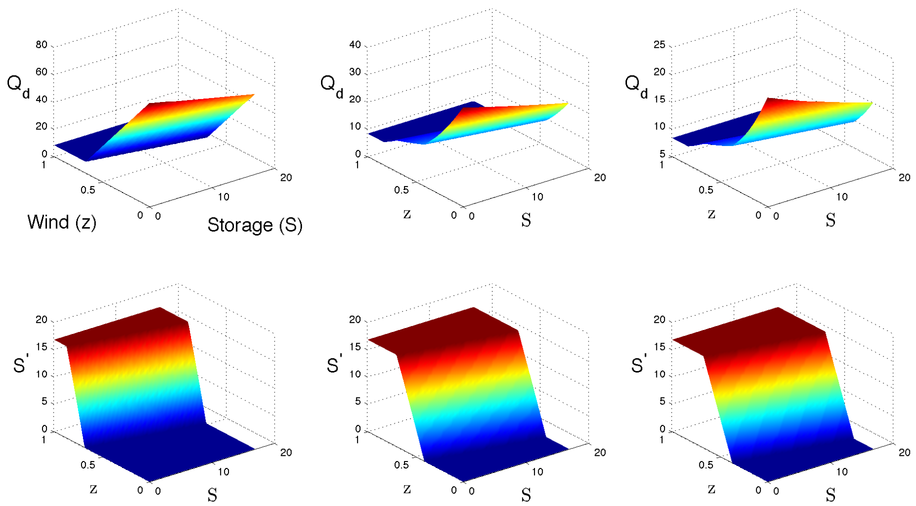
5 Simulation Results

Figure (3) presents the optimal decision rules for three different (linear, quadratic and cubic) cost functions. To be consistent with our earlier analysis, we present only the decision rules regarding the fossil fuel energy generation, $Q_d(S, z)$, and energy storage that will be transferred to the next period, $S'(S, z)$.

Considering the case with the linear cost function in generating the fossil fuel energy one can see that when the wind strength is highest, i.e., $z = 1$, and $z\bar{Q}_c = 100.8$GW, then it is optimal to generate the fossil fuel energy at its ramp-up level (see Figure (3a)-i). It is also optimal to store energy up to its capacity, 16.8GW, which is an outcome independent of the level of stored energy in this case (see Figure (3a)-ii). Furthermore, when the wind stregth is less than 0.5, all stored energy will then be consumed, which is a result independent of how much energy was transferred into the current period.

The optimal decision rules for the two remaining cases are quite distinct. Inline with Proposition 3.1, one can see that the costlier it gets to generate the fossil fuel energy, the lower the corresponding generation levels and the higher the level of energy transferred into the next period.[17] For example, if $z = .5$ and there is no stored energy, then $S' = (0, 5.2, 6.9)$ gigawatts for a constant-, increasing- and increasingly increasing-cost fossil fuel energy industry, respectively.

[17]For all variations of z and S, while the fossil fuel energy generation takes its lowest values, the energy levels transferred to the next period are the highest for a cubic cost function, i.e., $C_d''' > 0$.



(a) *Linear cost function* **(b)** *Quadratic cost function* **(c)** *Cubic cost function*

Figure 3: *Optimal decision rules for fossil fuel energy generation, Q_d, and energy storage, S', for different cost functions.*

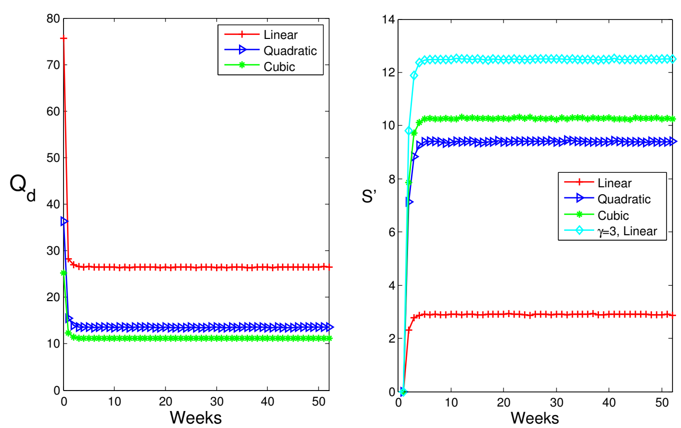
A lower level of stored energy for each pair of z and S when the cost function is linear can be attributed to the lower opportunity cost of not storing energy in the current period: if the wind power is low and energy is not stored, then, in case it is required, the cost of generating the required energy from fossil fuels will not be too costly. However, this is not necessarily the case when the cost function is nonlinear: if there is no stored energy and suddenly the wind ceases to blow, then the economy would have to incur greater costs to get the desired level of energy from fossil fuels.

Having solved for the optimal decision rules, we can examine the long-run tendencies of the model variables. Here, we aim at computing the steady state mean values for the model variables and analyze how they respond to different specifications of the cost function and model parameters.

In doing this we simulate the representative paths for the model variables using Monte Carlo simulations. Given that we work with a stationary distribution, i.e., that the transition probabilities are time invariant, we can argue that our problem possesses a steady state distribution so that we can calculate the steady state mean values for the variables we are interested in.

Assuming three different cost functions in generating fossil fuel energy, the results of the simulations are summarized by Figure 4a. As expected from the previous discussion regarding the optimal decision rule, the fossil fuel steady state (SS) mean levels are the smallest, approximately 10GW, for the case with the cubic cost function. On the contrary, the SS mean value for the stored energy is the highest, 10.2GW for the same case. Moreover, when one considers the long-run tendencies given that the cost structure of the fossil fuel energy industry is constant, i.e., a linear cost function, we see that the fossil fuel energy SS mean takes its highest value, 27GW, while the stored energy gets much lower, approximately 2GW. In line with Proposition 3.1, the simulation results show the impact a positive third-order derivative of the cost function can have on energy storage decisions.

Another fundamental result we got previously was the effect of prudence on precautionary energy storage decisions. In looking at the effect of a more prudent economy, we take $\gamma = 3$. The simulations show that a higher level of prudence can alter the results significantly. Compared to the previous cases with different cost structures, we see that a higher level of prudence can indeed result in a much higher level of SS energy savings, even if the cost function is linear (see Figure 4b).



(a) *Fossil fuel energy* **(b)** *Energy storage*

Figure 4: *Steady state analysis - mean values*

6 Capacities

In doing the analysis regarding the value of capacity increments in renewable energy, we make use of the steady state mean analysis, where we look at the long-run tendency –the expected paths– calculated from the optimal decision rule, $\{Q^*(S,z), Q_d^*(S,z), R^*(S,z), S'^*(S,z)\}$, and the various realizations of the random shocks. Under various scenarios, the Monte-Carlo simulations justify that such steady state mean values can be calculated (see Figure 4).

Given that we deal with a stationary process and there exists a steady state mean level for stored energy, μ^s, we can then show that the expected payoff at each period is the same using the law of iterated expectations.

In doing this, consider two value functions, one for the current period and one for the following one at the steady state: $\{V(\mu^s, z_t), V(\mu^s, z_{t+1})\}$. The Bellman equation can then be shown as:

$$V(\mu^s, z_t) = \max_{Q_{dt}} \left\{ U(Q_{dt} - z_t\bar{Q}_c - (1-\phi)\mu^s) - C_d(Q_{dt}) + \delta \mathbb{E}_{z_{t+1}} \left[V(\mu^s, z_{t+1}) \right] \right\} \tag{30}$$

Taking the expectation at time t gives:

$$\mathbb{E}_{z_t}[V(\mu^s, z_t)] = \max_{Q_{dt}} \left\{ \mathbb{E}_{z_t} \left[U(Q_{dt} - z_t\bar{Q}_c - (1-\phi)\mu^s) - C_d(Q_{dt}) \right] + \delta \mathbb{E}_{z_t} \left[\mathbb{E}_{z_{t+1}} \left[V(\mu^s, z_{t+1}) | z_t \right] \right] \right\} \tag{31}$$

We can write the second term on the RHS of (31) as:

$$\mathbb{E}_{z_t} \left[\mathbb{E}_{z_{t+1}} \left[V(\mu^s, z_{t+1}) | z_t \right] \right] = \mathbb{E}_{z_t} \left[\int_0^1 V(\mu^s, z_{t+1}) f_{z_{t+1}|z_t}(z_{t+1}|z_t) dz_{t+1} \right] \tag{32}$$

As the joint probability distribution, which, given the i.i.d property, can be shown as $f_{z_{t+1}|z_t}(z_{t+1}|z_t) = f_{z_{t+1}}(z_{t+1})$, we have:

$$\begin{aligned} \mathbb{E}_{z_t} \left[\mathbb{E}_{z_t+1} \left[V(\mu^s, z_{t+1}) | z_t \right] \right] =& \mathbb{E}_{z_t} \left[\int_0^1 V(\mu^s, z_{t+1}) f_{z_{t+1}}(z_{t+1}) dz_{t+1} \right] \\ =& \mathbb{E}_{z_t} \left[\int_0^1 V(\mu^s, z_t) f_{z_t}(z_t) dz_t \right] \\ =& \mathbb{E}_{z_t}[V(\mu^s, z_t)] \end{aligned} \tag{33}$$

Using this result in (31) one gets:

$$\mathbb{E}_{z_t}[V(\mu^s, z_t)] = \frac{1}{1-\delta} \max_{Q_{dt}} \left\{\mathbb{E}_{z_t}\left[U(Q_{dt} - z_t\bar{Q}_c - (1-\phi)\mu^s) - C_d(Q_{dt}))\right]\right\} \tag{34}$$

The max operator allows one to apply the Envelope Theorem. Taking the derivative wrt $\bar{Q}_c$ and iterating the resulting expression one period forward gives:

$$\mathbb{E}_{z_{t+1}}\left[\frac{\partial V(\mu^s, z_{t+1})}{\partial \bar{Q}_c}\right] = \frac{1}{1-\delta} \max_{Q_{dt+1}} \left\{\mathbb{E}_{z_{t+1}}\left[z_{t+1}U'(Q_{dt+1} - z_{t+1}\bar{Q}_c - (1-\phi)\mu^s)\right]\right\} \tag{35}$$

Assuming $Q_d(\mu^s, z_t) > \underline{Q}_d$ (i.e., at steady state the fossil fuel energy takes an interior value), taking the derivative of the value function, (30), wrt $\bar{Q}_c$ and substituting (35) to the resulting expression gives:

$$\frac{\partial V(\mu^s, z_t)}{\partial \bar{Q}_c} = z_t U'(Q(\mu^s, z_t)) + \frac{\delta}{1-\delta}\mathbb{E}_{z_{t+1}}\left[z_{t+1}U'(Q(\mu^s, z_{t+1}))\right] > 0 \tag{36}$$

From the FOC wrt Q_d we have $U'(Q_d) = C_d'(Q_d)$, which given the steady state mean value μ^s is:

$$U'(Q(\mu^s, z_t)) = C_d'(Q_d(\mu^s, z_t)) \tag{37}$$

Plugging this result in (36) one gets:

$$\frac{\partial V(\mu^s, z_t)}{\partial \bar{Q}_c} = z_t C_d'(Q_d(\mu^s, z_t)) + \frac{\delta}{1-\delta}\mathbb{E}_{z_{t+1}}\left[z_{t+1}C_d'(Q_d(\mu^s, z_{t+1}))\right] > 0 \tag{38}$$

The comparative statics wrt μ^s gives:

$$\frac{\partial Q_d(\mu^s, z_t)}{\partial \mu^s} = -(1-\phi)\frac{U''(Q(\mu^s, z_t)}{C_d''(Q_d(\mu^s, z_t)) - U''(Q(\mu^s, z_t)} > 0 \tag{39}$$

This result indicates that for $\mu^s > 0$, $Q_d(\mu^s, z_t) > Q_d(0, z_t)$, and hence, $C_d'(Q_d(\mu^s, z_t)) \geq C_d'(Q_d(0, z_t))$. From (38), this then implies:

$$\frac{\partial V(\mu^s, z_t)}{\partial \bar{Q}_c} > \frac{\partial V(0, z_t)}{\partial \bar{Q}_c}. \tag{40}$$

Expression (40) leads to the following proposition:

Proposition 6.1. The marginal gain from an increase in renewable energy capacity increases in energy storage.

The result follows from (40).

From (39), an increase in the steady state level of stored energy will be matched with an increase in the long-run level of fossil fuel energy when both fossil fuel energy and energy storage take values between their boundaries. Unless the cost function is linear, this will cause a higher cost of energy generation, i.e., a higher price for energy, which will then increase the revenues that will accrue to both the renewable energy operators and new capacities for renewable energy.

Corollary 6.2. If there is a constant cost fossil fuel energy industry, i.e., the cost function in generating the fossil fuel energy is linear, then changes in the level of stored energy will have no influence on the value of capacity increments in renewable energy.

For proof, see Appendix D.

In order to find the expression we make use of in calculating the value of a capacity increment in renewable energy, one can decompose equation (36) and get:

$$\begin{aligned}\frac{\partial V(\mu^s, z_t)}{\partial \bar{Q}_c} = z_t C_d'(Q_d(\mu^s, z_t)) + \frac{\delta}{1-\delta}\Big(&\mathrm{Cov}(z_{t+1}, C_d'\left(Q_d(\mu^s, z_{t+1})\right)) \\ &+ \mathbb{E}\left[z_{t+1}\right]\mathbb{E}\left[C_d'(Q_d(\mu^s, z_{t+1}))\right]\Big).\end{aligned} \tag{41}$$

Suppose that δ is close to 1, i.e., the future is heavily weighted. As the implicit weight on the current period approaches 0 when δ approaches 1 (i.e. if $\delta \to 1$, then $\delta/(1-\delta) \to \infty$), one can then disregard the current effect from a change in $\bar{Q}_c$ and get:

$$\frac{\partial V(\mu^s, z_t)}{\partial \bar{Q}_c} = \frac{\delta}{1-\delta}\Big(\mathrm{Cov}(z_{t+1}, C_d'\left(Q_d(\mu^s, z_{t+1})\right)) + \mathbb{E}\left[z_{t+1}\right]\mathbb{E}\left[C_d'(Q_d(\mu^s, z_{t+1}))\right]\Big), \tag{42}$$

Taking the expectation of a second-order Taylor approximation for $C_d'(Q_d(\mu^s, z_{t+1}))$

around $\mathbb{E}[z]$ then gives:

$$\frac{\partial V(\mu^s, z_t)}{\partial \bar{Q}_c} = \frac{\delta}{1-\delta}\Bigg(\mathrm{Cov}\Big(z_{t+1}, C'_d\left(Q_d(\mu^s, z_{t+1})\right)\Big) + \mathbb{E}\left[z\right]\left(C'_d\left(Q_d(\mu^s, \mathbb{E}[z])\right) + \frac{1}{2}g''(\mathbb{E}[z])\sigma_z^2\right)\Bigg), \tag{43}$$

where $g''(\cdot)$ is given by (14). The expression given by (43) is the one we make use of in calculating the value of a capacity increment in renewable energy.

One can see from (43) that there are two opposing effects. The first term in the parenthesis, which is the covariance between the fossil fuel energy and the weather condition, is negative: as we have shown earlier, a higher z, hence a higher level of renewable energy, causes a lower level of energy generated from fossil fuels. The second term in the parenthesis, which is the product of the expected value for z and the marginal cost of fossil fuel energy at the steady state level for stored energy, is positive. From this second term, one can see that a higher degree of prudence, convexity in the marginal cost for fossil fuel energy and volatility in the weather conditions induce the value of renewable energy capacity increments positively.

7 Capacities - Simulation Results

In order to make some real life comparisons, we convert UoN (units of the numeraire, see p. 18) to US dollars. In calculating the dollar value of UoN, we use the estimated average levelized cost of new generation coal power plants entering service in 2018 and pick the average operating and management (O&M) cost of a conventional coal power plant: 29.2\$/MW (EIA, 2013). Converting this value to GW per week gives \$4.9056m. Thus, for $c_d = 0.000175$UoN the numeraire is worth \$28.0368bn. As we did previously, we take an annual discount rate of 5 percent, which corresponds to a weekly discount factor of $\delta = 0.9991$.

Making use of (43) and the dollar value of the numeraire, the net present values for capacity increments for the constant cost and increasing cost fossil fuel energy industries are presented on Table 1.

Assuming a quadratic cost function, hence an increasing cost industry (ICI) for the fossil fuel energy generation, the results show that in the absence of a storage technology the value a 1MW/h wind turbine is expected to create in its lifetime is \$3.03m (0.1082UoN). An investor can then compare this to the cost of a 1MW/h wind turbine in order to make a comparison. In the presence of the storage technology for energy, this value jumps to \$3.51m (0.1251UoN) (approximately 16% higher value), which is a result in line with Proposition 6.1.

Table 1: *Value of* 1*MWh windmill in the long run. ICI and CCI stand for increasing-cost and constant-cost fossil fuel industries*

Cost structure	ICI	CCI
Energy Storage	\$3.51m	\$2.62m
No Energy Storage	\$3.03m	\$2.62m

One can also see in Table 1 that for a linear cost function, hence a constant cost industry (CCI), an additional wind turbine is worth the same, \$2.62m, regardless of the level of stored energy, which is a result that is inline with Corollary 6.2.

We conclude this section with a word of precaution concerning the discount rate and the maximum renewable energy capacity employed in the simulations. As can be seen from Appendix E.2, the value of capacity increments in the renewable energy can be sensitive to the level of the discount rate used. Moreover, the same value will also change with respect to the level of the existing renewable capacity (see Appendix E.3). However, determining the right discount rate and the capacity to use in the simulations are beyond our scope here and can be explored in depth in a separate paper.

8 Conclusion and Discussion

In line with the global efforts to reduce CO_2 emissions, renewables have an extensive potential to substitute for the fossil fuels. However, they also have their shortcomings. One of them, maybe the most crucial one, is the intermittency problem that can jeopardize immediate access to energy. One technology considered to alleviate, or even cause the intermittency problem to be negligible, is energy storage. Yet the economics of energy generation lacks the treatment of intertemporal welfare decisions in the presence of intermittent renewable energy and energy storage technology. This may become a serious drawback, as without taking this into account, long-term analysis and the policy decisions in this respect can be biased and even misleading.[18]

By approaching the problem both analytically and numerically, we attempt to fill this gap. Our analytical results show the conditions where prudence can have considerable effect on energy storage decisions. We also show how the cost structures, including the third-order derivative of the cost function in generating the fossil fuel energy, can influence energy storage decisions.

Using numerical simulations, we then calculate the optimal decision rule, i.e., optimal policy functions, which are vital in navigating decisions regarding how much energy to generate from fossil fuels and how much to use from stored energy (or how much to store). We use this policy tool to analyze the variables' long-term tendencies, i.e., steady state mean levels. Our analytical and numerical results also show that capacity increments are desired more when the economy is expected to store a higher level of energy.

Our results not only reveal that prudence and a third-order derivative of the cost function are important for energy storage decisions, but also show that a prior knowledge of the prudence level and the cost-structure of the fossil fuel industry can be quite fundamental in the optimal management of energy sources and the evaluation of renewable energy investments.

Our study can be extended in several directions. First, one can extend the current model by taking into account investment decisions in capacities. It is also interesting to incorporate a climate module and investigate the effects of climate change and hence the climate policies on the use of fossil fuels, intermittent renewable energy and energy storage. One can also consider R&D investments and technological change and analyze how the use of different energy sources and their technologies evolve over time depending on both climate and R&D policies. Last but not least, a further investigation of the effects of prudence and the cost structures on the economic decisions can be quite important not

[18] It is also important to note that the long-term policy suggestions of assessment models need be taken with a grain of salt not only because they are big abstractions of complex dynamics, but also the intermittency problem (thus, shorter time periods) and with it the energy storage decisions are excluded. This can have cogent influence on the ongoing research in assessment modeling and climate change, as their calculations and conclusions extend to the near and distant futures.

only in the literature in energy economics, but also in the literature on prudence in general. The decentralization of the optimal allocation decisions by market mechanisms and the investigation of how allocations are modified when risk attitudes and time preferences change is another interesting avenue, which we pursue in another paper.

References

Ambec, S. and C. Crampes (2012). Electricity provision with intermittent sources of energy. *Resource and Energy Economics 34*(3), 319–336.

Arvizu, D., P. Balaya, L. Cabeza, T. Hollands, A. Jäger-Waldau, M. Kondo, C. Konseibo, V. Meleshko, W. Stein, Y. Tamaura, and R. Z. H. Xu (2011). Direct Solar Energy. In: IPCC Special Report on Renewable Energy Sources and Climate Change Mitigation [O. Edenhofer, R. Pichs-Madruga, Y. Sokona, K. Seyboth, P. Matschoss, S. Kadner, T. Zwickel, P. Eickemeier, G. Hansen, S. Schlömer, C. von Stechow (eds)]. Technical report, IPCC, Cambridge, United Kingdom and New York, NY, USA.

Crampes, C. and M. Moreaux (2001). Water resource and power generation. *International Journal of Industrial Organization 19*(6), 975–997.

Crampes, C. and M. Moreaux (2010). Pumped storage and cost saving. *Energy Economics 32*(2), 325–333.

EIA (2011). International Energy Outlook 2011. Technical report, U.S. Energy Information Administration.

EIA (2013). Annual Energy Outlook 2013 - Levelized Cost of New Generation Resources. Technical report, EIA.

EIU (2013). The Uganda Energy Opportunity. `https://globalconnections.hsbc.com/us/en/articles/uganda-energy-opportunity`. Accessed: 2013-08-30.

Førsund, F. R. (2007). *Hydropower Economics.* Springer, New York, USA.

Forsund, F. R. (2012). Pumped-storage hydroelectricty. *CREE Working paper* (14).

Førsund, F. R. and L. Hjalmarsson (2011). Renewable energy expansion and the value of balance regulation power. *Modern Cost-benefit Analysis of Hydropower Conflicts. Edward Elgar Publishing*, 97–126.

Heal, G. (2009). Climate economics: a meta-review and some suggestions for future research. *Review of Environmental Economics and Policy 3*(1), 4–21.

IPCC (2013). Summary for Policymakers. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Technical report, IPCC, Cambridge, United Kingdom and New York, NY, USA.

Judd, K. L. (1992). Projection methods for solving aggregate growth models. *Journal of Economic Theory, 58*, 410–452.

Judd, K. L. (1998). *Numerical Methods in Economics.* Yhe MIT Press, Massachusetts, London, England.

Kaldellis, J., A. Gkikaki, E. Kaldelli, and M. Kapsali (2012). Investigating the energy autonomy of very small non-interconnected islands: A case study: Agathonisi, Greece. *Energy for Sustainable Development 16*(4), 476–485.

Kanakasabapathy, P. (2013). Economic impact of pumped storage power plant on social welfare of electricity market. *Electrical Power and Energy Systems 45*, 187–193.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica: Journal of the Econometric Society*, 53–73.

Koopmans, T. C. (1958). *Water storage policy in a simplified hydroelectric system.* Cowles Foundation for Research in Economics at Yale University.

Korpaas, M., A. T. Holena, and R. Hildrumb (2003). Operation and Sizing of Energy Storage for Wind Power Plants in a Market System. *Electrical Power and Energy Systems 25*, 559–606.

Leland, H. E. (1968). Saving and uncertainty: The precautionary demand for saving. *The Quarterly Journal of Economics 82*(3), 465–473.

Miranda, M. J. and P. L. Fackler (2002). *Applied Computational Economics and Finance.* MIT Press.

Sandmo, A. (1970). The effect of uncertainty on saving decisions. *The Review of Economic Studies*, 353–360.

Stokey, N. L. (1989). *Recursive Methods in Economic Dynamics*. Harvard University Press.

Tuohy, A. and M. O'Malley (2011). Pumped storage in systems with very high wind penetration. *Energy Policy 39*(4), 1965–1974.

Van de Ven, P., N. Hegde, L. Massoulie, and T. Salonidis (2011). Optimal control of residential energy storage under price fluctuations. In *ENERGY 2011, The First International Conference on Smart Grids, Green Communications and IT Energy-aware Technologies*, pp. 159–162.

APPENDIX

A Blackwell's sufficient conditions for a contraction

Theorem A.1. *(Blackwell's sufficient conditions for a contraction) Let* $\mathrm{X} \subseteq \mathbb{R}^l$*, and let* $\mathrm{B(X)}$ *be a space of bounded functions* $f : \mathrm{X} \to \mathbb{R}$*, with supremum norm* $\|\cdot\|_\infty$*. Let* $T : \mathrm{B(X)} \to \mathrm{B(X)}$ *be an operator satisfying*

1. *(Monotonicity) for* $f, g \in \mathrm{B(X)}$ *and* $f(\mathrm{x}) \le g(\mathrm{x})$, $\forall \mathrm{x} \in \mathrm{X}$*, implies* $(Tf)(\mathrm{x}) \le (Tg)(\mathrm{x})$, $\forall \mathrm{x} \in \mathrm{X}$*;*

2. *(Discounting) there exists some* $\delta \in (0,1)$ *such that*

$$[T(f+a)](\mathrm{x}) \le (Tf)(\mathrm{x}) + \delta a, \text{all } f \in \mathrm{B(X)}, a \ge 0, \mathrm{x} \in \mathrm{X}.$$

Then T *is a contraction with modulus* δ.[19]

[19] $(f+a)(\mathrm{x})$ is the function defined by $(f+a)(\mathrm{x}) = f(\mathrm{x}) + a$. For the proof we refer the reader to Stokey (1989).

B Application of Blackwell's sufficient conditions for a contraction to the model

Proof. Looking at the equation of motion for stored energy, S, one can see that it takes its maximum value when energy consumption is null and $z = 1$: $S^{\max} = (Q_d + Q_c)/(1 - \phi)$. This defines the state space $\mathrm{X} \subseteq [0, \bar{Q}_d + \bar{Q}_c] \subseteq \mathbb{R}$ and B(X) the function space of the bounded functions $f : \mathrm{X} \to \mathbb{R}$ with supremum norm.

In the energy storage problem, we defined an operator T by:

$$(Tv)(S,z) = \max_{\{Q,Q_d,Q_c,R,S'\}} \left\{ U(Q) - c_d(Q_d) - c_c(\bar{Q}_c) + \delta \mathbb{E}_z \left[v(S',z') \right] \right\}$$

If $v(S',z') \leq \hat{v}(S',z')$ for all values of S', then the objective function for which $T\hat{v}$ is the maximized value is uniformly higher than the function for which Tv is the maximized value, which makes the monotonicity requirement obvious. The discounting requirement is also satisfied from the following:

$$\begin{aligned}
(T(v+a))(S,z) &= \max_{\{Q,Q_d,Q_c,R,S'\}} \left\{ U(Q) - c_d(Q_d) - c_c(\bar{Q}_c) + \delta \mathbb{E}_z \left[v(S',z') + a \right] \right\} \\
&= \max_{\{Q,Q_d,Q_c,R,S'\}} \left\{ U(Q) - c_d(Q_d) - c_c(\bar{Q}_c) + \delta \mathbb{E}_z \left[v(S',z') \right] \right\} + \delta a \\
&= (Tv)(S,z) + \delta a
\end{aligned}$$

□

C Numerical implementation of the model

C.1 Method description

We solve the dynamic stochastic decision problem by collocation method. In doing this we approximate the value function by an approximant $\widetilde{V}(S)$ that is parameterized by and solved for a vector of parameters, $\boldsymbol{\beta}$.

Abstracting from intermittency, z, a function can be approximated by a combination of n linearly independent basis functions, $\{\psi_i\}_{i=0}^{n}$, and basis coefficients, $\{\beta\}_{i=0}^{n}$, where n represents the number of collocation points:

$$F(x) \approx \widetilde{F}(x) = \sum_{i=1}^{n} \beta_i \psi_i(x).$$

The interpolation problem in one dimension is then to find $\{\beta\}_{i=0}^{n}$ such that it satisfies the function F at n interpolation points.

In vector notation this can be written as the following:

$$F(\boldsymbol{x}) = \boldsymbol{\psi}(x)\boldsymbol{\beta},$$

where $\boldsymbol{\Psi(x)} = [\psi_1(x)\ \psi_2(x)\ \psi_3(x)\ \ldots\ \psi_{n+1}(x)]$ is the Chebyshev Vandermonde matrix, $\boldsymbol{\beta} = [\beta_1\ \beta_2\ \beta_3\ \ldots\ \beta_{n+1}]'$ and $\boldsymbol{x} = [x_1\ x_2\ ...\ x_{n+1}]'$,

$$\boldsymbol{\Psi}(\boldsymbol{x}) = \begin{bmatrix} \psi_1(x_1) & \psi_2(x_1) & \psi_3(x_1) & \ldots & \psi_{n+1}(x_1) \\ \psi_1(x_2) & \psi_2(x_2) & \ldots & \ldots & \psi_{n+1}(x_2) \\ \psi_1(x_3) & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \psi_1(x_{n+1}) & \ldots & \ldots & \ldots & \psi_{n+1}(x_{n+1}) \end{bmatrix}.$$

Similarly, in approximating a value function, we search for a coefficient vector, $\boldsymbol{\beta}$, that ensures that the approximant satisfies the Bellman equation and the equilibrium conditions at the n collocation nodes (one can think of collocation nodes as discrete "states of the economy").

In our energy consumption and storage problem, we approximate a bivariate function, $V(S, z)$, as the planner considers the amount of stored energy and weather conditions before taking decisions. Therefore, we need apply the collocation method solution strategy in a multidimensional setting (i.e., multidimensional interpolation).

We solve (3) which is simplified to give:[20]

$$V(S,z) = \max_{\{Q_d, S'\}} \left\{ \frac{\left(Q_d + z\bar{Q}_c + \phi S - S'\right)^{1-\gamma}}{1-\gamma} - c_d\left(Q_d\right) + \delta \mathbb{E}_z \left[V(S',z')\right] \right\}$$
$$\text{s.t} \quad 0 \leq S \leq \bar{S},$$
$$\underline{Q}_d \leq Q_d \leq \bar{Q}_d.$$

We approximate the value function using Chebyshev polynomials.[21] In doing this we discretize z into K (z_k for $k = (1,2,...K)$) and S into n collocation nodes (S_i for $i = (1,2,...,n)$). We determine the basis function coefficients for each z and S. For n basis functions, there are going to be n basis coefficients, and given K different weather states, the computational problem is to solve for $K \cdot n$ coefficients. Let us denote these coefficients by $\boldsymbol{\beta} = [\boldsymbol{\beta}_1 \ \boldsymbol{\beta}_2 \ \ldots \ \boldsymbol{\beta}_K]$, where $\boldsymbol{\beta}_z = [\beta_{1,z} \ \beta_{2,z} \ \ldots \ \beta_{n,z}]'$.

For each state of the weather, z_k, and for each level of stored energy, S_i, the approximant is formed as follows:

$$V(S_i, z) \approx \widetilde{V}(S_i, z) = \sum_{j=1}^{n} \beta_{j,z} \psi_j(S_i)$$

Given $V(S_i, z)$, we need to form the approximant to $V(S'_i, z'_k)$ as well. In doing this for S_i and z_k, we need to compute the level for the stored energy in the period ahead, S', and energy generation today Q_d given the intervals $\bar{S} \leq S \leq \bar{S}$ and $\underline{Q}_d \leq Q_d \leq \bar{Q}_d$. Using these boundaries we construct a grid for fossil fuel energy and energy storage, $\{Q_{di,z_k,l}\}_{l=1}^{n}$ and $\left\{S'_{i,z_k,l}\right\}_{l=1}^{n}$:

$$\boldsymbol{Q}_{di,z_k} = \{Q_{di,z_k,1} \ Q_{di,z_k,2} \ \ldots \ Q_{di,z_k,n}\}$$
$$\boldsymbol{S}'_{i,z_k} = \{S'_{i,z_k,1} \ S'_{i,z_k,2} \ \ldots \ S'_{i,z_k,n}\}$$

Given the approximants of the value function, we have $(K \cdot n)$ equations in

[20] The optimization here is done w.r.t Q_d and S' instead of Q, S'. This is due to the need for assigning boundary values for fossil fuel energy generation and energy storage in the numerical calculations.

[21] Chebychev basis polynomials in combination with Chebychev interpolation nodes can yield extremely well-conditioned interpolation collocation equations that one can accurately and efficiently solve. For a discussion regarding Chebychev basis functions and nodes, we refer the reader to Judd (1992), Judd (1998) and Miranda and Fackler (2002).

$(K \cdot n)$ unknowns:

$$\sum_{j=1}^{n} \beta_{j,z} \psi_j(S_i) = \max_{\{Q_d, S'\}} \left\{ \frac{\left(Q_{di,z_k,l} + z\bar{Q}_c + \phi S - S'\right)^{1-\gamma}}{1-\gamma} - c_d\left(Q_d\right) \right.$$
$$\left. + \delta \sum_{k=1}^{K} \sum_{l=1}^{n} \omega_k \beta_{j,z_k} \psi_j(S'_{i,z_k,m}) \right\}_{l=1}^{n}$$

where in approximating the integral operation we replaced the continuous random variable z_k with its discrete counter part ω_k, the weight functions, ω_k, using Gaussian quadrature scheme.[22] The weight functions are defined over the interval K. Quadrature nodes, here z_k, for $k = \{1, 2, ...K\}$, and corresponding quadrature weights ω_k, for $k = \{1, 2, ...K\}$ are selected such that $2K$ moments are satisfied.[23]

Above, we showed the approximant for $V(S'_i, z'_k)$ in its explicit form:

$$V(S'_i, z'_k) \approx \widetilde{V}(S'_i, z'_k) = \sum_{j=1}^{n} \beta_{j,z_k} \psi_j(S'_{i,z_k,l})$$
$$= \sum_{j=1}^{n} \beta_{j,z_k} T_{j-1} \left[2 \left(\frac{S'_{i,z_k,l} - \underline{S}}{\bar{S} - \underline{S}} \right) - 1 \right] \text{ for } l = \{1, 2, \ldots, n\}$$

where $\boldsymbol{\psi}_j(S'_{i,z_k}) = T_{j-1} \left[2 \left(\frac{S'_{i,z_k,l} - \underline{S}}{\bar{S} - \underline{S}} \right) - 1 \right]$ are the Chebyshev polynomial basis functions.

Having explained how the polynomial interpolation can work, we now explain the procedure of how to calculate the basis function coefficients, $\boldsymbol{\beta} = [\boldsymbol{\beta}_1 \ \boldsymbol{\beta}_2 \ \ldots \ \boldsymbol{\beta}_K]$. First we need to make a guess for the initial values of the basis functions' coefficients: $\boldsymbol{\beta}^0 = [\boldsymbol{\beta}_1^0 \ \boldsymbol{\beta}_2^0 \ \ldots \ \boldsymbol{\beta}_K^0]$. We then need to construct a grid of Chebyshev nodes, $\boldsymbol{u}_{n \times 1}$, and convert them into grid of stored energy, $\boldsymbol{S}$. The

[22] For a weight function defined on an interval K, $\int_K z\omega(z)dz \simeq \sum_{k=1}^{K} \omega_k z_k$.

[23] For the intermittent renewable energy production, $Q_c = z\bar{Q}_c$, its expected value can be calculated numerically as follows:

$$\mathbb{E}\left[Q_c\right] = \int_K z\bar{Q}_c \omega(z) dz \approx \sum_{k=1}^{K} \omega_m z_m \bar{Q}_c$$

mapping looks like the following:

$$\boldsymbol{u} \to \boldsymbol{S} \in [\underline{S}, \bar{S}],\ \boldsymbol{S} = \frac{\bar{S} + \underline{S}}{2}\boldsymbol{I} + \frac{\bar{S} - \underline{S}}{2}\boldsymbol{u}$$

where $\boldsymbol{I}$ is a vector of ones, $\boldsymbol{I}_{n\times 1}$.

For $k = \{1, 2, \ldots, K\}$ and i={1,2,...,n}, we construct a feasible grid of energy generation Q_d and S_i' using Chebyshev nodes:

$$\begin{aligned}
\boldsymbol{u} \to \boldsymbol{Q}_d \in [\underline{Q}_d, \bar{Q}_d],\ \boldsymbol{Q}_{di,z_k} &= \frac{\bar{Q}_d + \underline{Q}_d}{2}\boldsymbol{I} + \frac{\bar{Q}_d - \underline{Q}_d}{2}\boldsymbol{u} \\
\boldsymbol{u} \to \boldsymbol{S}' \in [\underline{S}', \bar{S}'],\ \boldsymbol{S}'_{i,z_k} &= \frac{\bar{S}}{2}\left(\boldsymbol{I} + \boldsymbol{u}\right)
\end{aligned}$$

where the last equality resulted from $\underline{S} = 0$.

For $\boldsymbol{S}'$, we have the Chebyshev Vandermonde matrix: $\boldsymbol{\Psi}(\boldsymbol{S})$. Then

$$\widetilde{V}(\boldsymbol{S}, \boldsymbol{z}) = \frac{\left(\boldsymbol{Q}_{d,z} + \boldsymbol{z}\bar{Q}_c + \phi\boldsymbol{S} - \boldsymbol{S}'\right)^{1-\gamma}}{1-\gamma} - c_d\left(\boldsymbol{Q}_{d,z}\right) + \delta\sum_{k=1}^{K}\omega_k \boldsymbol{\Psi}(\boldsymbol{S}')\boldsymbol{\beta}_k^0$$

Taking the maximal entries in $\widetilde{V}(\boldsymbol{S}, \boldsymbol{z})$ we can construct $\tilde{\boldsymbol{V}}(\boldsymbol{\beta}^0)$ and update the coefficients according to Newton-Raphson method (see Judd (1998)):[24]

$$\boldsymbol{\beta}' = \boldsymbol{\beta} - \left[\boldsymbol{\Psi} - \widetilde{\boldsymbol{V}}^j(\boldsymbol{\beta})\right]^{-1}\left[\boldsymbol{\Psi}\boldsymbol{\beta} - \widetilde{\boldsymbol{V}}(\boldsymbol{\beta})\right]$$

where $\widetilde{\boldsymbol{V}}^j$ is the Jacobian of the approximant. One can then use the iterative update rule until the following difference gets to or smaller than a predetermined tolerance level, ϵ:

$$\boldsymbol{\beta}' - \left(\boldsymbol{\beta} - \left[\boldsymbol{\Psi} - \widetilde{\boldsymbol{V}}^j(\boldsymbol{\beta})\right]^{-1}\left[\boldsymbol{\Psi}\boldsymbol{\beta} - \widetilde{\boldsymbol{V}}(\boldsymbol{\beta})\right]\right) < \epsilon.$$

Long-run analysis After solving for the collocation coefficients, $\boldsymbol{\beta}$, we can estimate the evolution of the variables in the model. Using the grid we constructed for the stored energy $\boldsymbol{S}$, the solution to the model gives us an implicit policy rule: $\boldsymbol{S}' = g(\boldsymbol{S}, \boldsymbol{z})$.[25]

[24]Where $\boldsymbol{\beta}^0$ was a guess for the initial values of the basis functions' coefficients

[25]Given S_i and z_k we now know what $S'_{i,k}$ is.

By satisfying the convergence criteria, we also solve for $\boldsymbol{S'}$. We can use these values to estimate the policy (transition) rule, hence solve for the Chebyshev function coefficients, $\boldsymbol{\phi}$:

$$\boldsymbol{S'} = \boldsymbol{\Psi}\boldsymbol{\phi} \rightarrow \boldsymbol{\phi} = \left(\boldsymbol{\Psi'}\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi'}\boldsymbol{S'}$$

Using these coefficients one can pick a random sequence for weather conditions z_t for $t = 1, 2, \ldots, T$. One can then generate another sequence for S':

$$S_{t+1} = \boldsymbol{\Psi}(S_t)\boldsymbol{\phi}$$

Suppose that we do this N times (for N large) by generating N pseudorandom sequences for z.[26] Given the policy functions we calculated, $S'(S, z)$ and $Q_d(S, z)$, and the initial states S_0 and z_0, we can then generate a representative path from the N paths. Calculating the average value from the various pseudorandom sequences, one would get representative paths for the model variables in the long run. We will call this procedure a Monte Carlo Simulation.

C.2 Numerical implementation

We solve dynamic programming equation (2) by using collocation method and update the collocation coefficients according to the Newton's method (see Appendix C.1).[27] We construct a 40 Chebychev polynomial basis functions by forming 40 collocation nodes (4 nodes along S and 10 nodes along z dimension) and 40 basis function coefficients. The Beta distribution for the intermittent wind is approximated by Gauss-Legendre quadrature with 20 nodes.

The code is written in Matlab. We use CompEcon toolbox described in (Miranda and Fackler, 2002) in generating and evaluating the Chebychev polynomials, and doing the Monte Carlo simulations.

[26]Pseudorandom sequences are sequences that display some properties satisfied by random variables, such as zero serial correlation and correct frequency of runs, although none satisfy all properties of an i.i.d random sequence (Judd, 1992).

[27]The predetermined tolerance level for the convergence criterion 1 x 10^{-7}.

D Proof of Corollary 6.2

Proof. In order to see this, one can replace $C_d'(Q_d(\bar{S}, z_t))$ with c_d (see equation (23)) in (38) and get:

$$\frac{\partial V(\bar{S}, z_t)}{\partial \bar{Q}_c} = z_t c_d + \frac{\delta}{1-\delta} c_d \mathbb{E}_{z_{t+1}}\left[z_{t+1}\right],$$

which is a constant. □

E Sensitivity Analysis

As the choice of the parameters, ϕ, the energy loss rate and γ, the coefficient of relative risk aversion, can have significant effects on the results, it is worth examining how changes in these parameters affect storage decisions. In doing the analysis, the results are based on the case with $\bar{Q}_c > Q^*$ and increasing-cost FFE generation industry ($\bar{Q}_c = 108.8$GW,$\bar{Q}_d = 100.8$GW, $\underline{Q}_d = 8.4$GW, $\bar{S} = 16.8$GW, $\underline{S} = 0$, $\gamma = 2$, $\delta = .98$).

E.1 Round-trip efficiency parameter: ϕ

From Figure 5 one can see that all scenarios discern the same pattern and display similar qualitative features. i.e., in the first few periods energy is accumulated and stays roughly on its long-run expected level. However, the lower the round-trip efficiency parameter is, the smaller is the room for energy storage, i.e., the lower levels of ϕ imply less enthusiastic storage policies. As a result, for $\phi \leq 0.4$ energy storage becomes suboptimal.

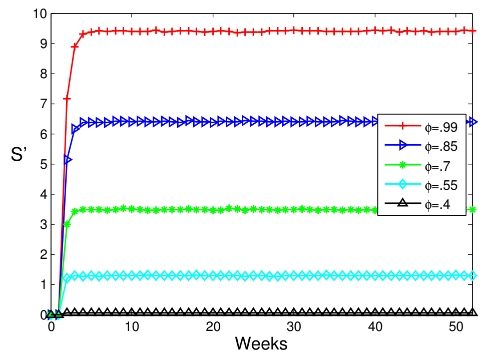


Figure 5: *Sensitivity analysis for round-trip efficiency parameter,* ϕ

E.2 Discount rate

As the choice of the discount rate parameter can play a crucial role in the simulations, it is worth examining how a change in this parameter can affect the value in renewable energy (RE) capacity increments. Originally, the discount rate that was employed in the simulations was %5. In this section, we explore a wider range for it, $\rho = \{.02, .03, .04, .05, .06, .07, .08, .09, .1\}$. Figure 6 summarizes the results.

One can see that for $\rho = .02$, the value of a 1MW/h capacity increment in the renewable energy is worth approximately \$8.5m, which then drops to \$3.51m when $\rho = .05$. Lastly, for $\rho = .1$, this value gets below \$2m. Although it is evident that our calculations of the net present value of capacity increments are sensitive to the discount rate, considerations regarding the appropriate choice of the discount rate is beyond the scope of the our paper.

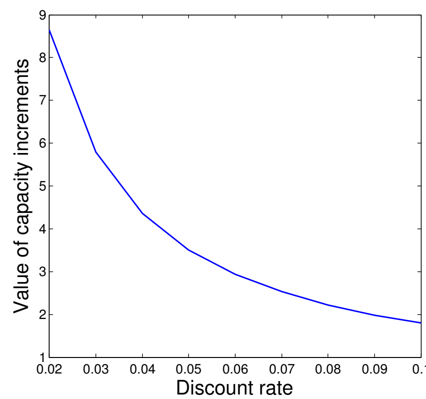


Figure 6: *Value of* 1MW/h *renewable energy (RE) capacity increments for different discount rates*

E.3 Renewable energy capacity

From Figure 7 it can be seen that the net present value of RE capacity increments change with respect to the available capacity. For a rather small capacity of RE (here, 50MW/h), the net present value of a 1MW/h capacity increment is $6.27m. As the fossil fuel energy would be generated at the ramp-up level of 50MW/h for rather big capacities of RE, this value then drops to and stabilizes at $2.6m.

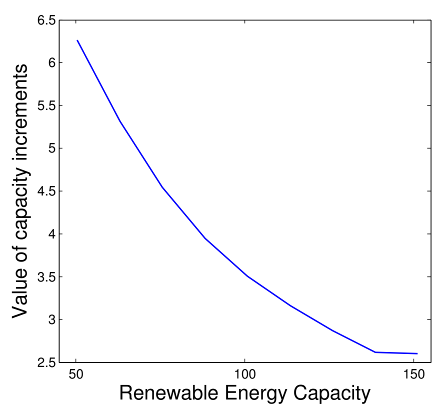


Figure 7: *Value of* 1MW/h *renewable energy (RE) capacity increments given available RE capacities*

Issued in the series Discussion Papers 2013

2013

01/13 January, **Lukáš Lafférs**, "Identification in Models with Discrete Variables".

02/13 January, **Ingvild Almås**, Anders Kjelsrud and Rohini Somanathan, "A Behaviour-based Approach to the Estimation of Poverty in India".

03/13 February, **Ragnhild Balsvik** and Line Tøndel Skaldebø, "Guided through the `Red tape'? Information sharing and foreign direct investment".

04/13 February, **Sissel Jensen**, Ola Kvaløy, Trond E. Olsen, and **Lars Sørgard**, "Crime and punishment: When tougher antitrust enforcement leads to higher overcharge".

05/13 February, **Alexander W. Cappelen**, **Trond Halvorsen**, **Erik Ø. Sørensen**, and **Bertil Tungodden**, "Face-saving or fair-minded: What motivates moral behavior?"

06/13 March, **Jan Tore Klovland** and **Lars Fredrik Øksendal**, "The decentralised central bank: regional bank rate autonomy in Norway, 1850-1892".

07/13 March, **Kurt Richard Brekke**, Dag Morten Dalen, and Tor Helge Holmås, "Diffusion of Pharmaceuticals: Cross-Country Evidence of Anti-TNF drugs".

08/13 April, **Kurt R. Brekke**, Luigi Siciliani, and Odd Rune Straume, "Hospital Mergers:A Spatial Competition Approach".

09/13 April, **Liam Brunt** and Edmund Cannon, "The truth, the whole truth, and nothing but the truth: the English Corn Returns as a data source in economic history, 1770-1914".

10/13 April, **Alexander W. Cappelen**, **Bjørn-Atle Reme**, **Erik Ø. Sørensen**, and **Bertil Tungodden,** "Leadership and incentives".

11/13 April, Erling Barth, **Alexander W. Cappelen**, and Tone Ognedal, "Fair Tax Evasion".

12/13 June, **Liam Brunt** and Edmund Cannon, "Integration in the English wheat market 1770-1820".

13/13 June, **Fred Schroyen** and **Nicolas Treich**, "The Power of Money: Wealth Effects in Contests".

14/13 August, **Tunç Durmaz** and **Fred Schroyen**, "Evaluating Carbon Capture and Storage in a Climate Model with Directed Technical Change".

15/13 September, **Agnar Sandmo**, "The Principal Problem in Political Economy: Income Distribution in the History of Economic Thought".

16/13 October, **Kai Liu**, "Health Insurance Coverage for Low-income Households: Consumption Smoothing and Investment".

17/13 December, **Øivind A. Nilsen**, **Lars Sørgard**, and **Simen A. Ulsaker**, "Upstream Merger in a Successive Oligopoly: Who Pays the Price?"

18/13 December, Erling Steigum and **Øystein Thøgersen**, "A crisis not wasted – Institutional and structural reforms behind Norway's strong macroeconomic performance".

2014

01/14 January**, Kurt R. Brekke**, Tor Helge Holmås, and Odd Rune Straume, "Price Regulation and Parallel Imports of Pharmaceuticals".

02/14 January, **Alexander W. Cappelen, Bjørn-Atle Reme, Erik Ø. Sørensen,** and **Bertil Tungodden, "**Leadership and incentives".

03/14 January, **Ingvild Almås, Alexander W. Cappelen, Kjell G. Salvanes, Erik Ø. Sørensen**, and **Bertil Tungodden**, "Willingness to Compete: Family Matters".

04/14 February, **Kurt R. Brekke**, Luigi Siciliani, and Odd Runde Straume, "Horizontal Mergers and Product Quality".

05/14 March, **Jan Tore Klovland**, "Challenges for the construction of historical price indices: The case of Norway, 1777-1920".

06/14 March, Johanna Möllerström, **Bjørn-Atle Reme**, and **Erik Ø. Sørensen**, "Luck, Choice and Responsibility".

07/14 March, Andreea Cosnita-Langlais and **Lars Sørgard**, "Enforcement vs Deterrence in Merger Control: Can Remedies Lead to Lower Welfare?"

08/14 March, **Alexander W. Cappelen, Shachar Kariv, Erik Ø. Sørensen**, and **Bertil Tungodden**, «Is There a Development Gap in Rationality?"

09/14 April, **Alexander W. Cappelen**, Ulrik H. Nielsen, **Bertil Tungodden**, Jean-Robert Tyran, and Erik Wengström, "Fairness is intuitive".

10/14 April, **Agnar Sandmo**, "The early history of environmental economics".

11/14 April, **Astrid Kunze**, "Are all of the good men fathers? The effect of having children on earnings".

12/14 April, **Agnar Sandmo**, "The Market in Economics: Behavioural Assumptions and Value Judgments".

13/14 April, **Agnar Sandmo**, "Adam Smith and modern economics".

14/14 April, Hilde Meersman, **Siri Pettersen Strandenes**, and Eddy Van de Voorde, "Port Pricing: Principles, Structure and Models".

15/14 May, **Ola Honningdal Grytten**, "Growth in public finances as tool for control: Norwegian development 1850-1950"

16/14 May, **Hans Jarle Kind**, Tore Nilssen, and **Lars Sørgard**, "Inter-Firm Price Coordination in a Two-Sided Market".

17/14 May, **Stig Tenold**, "Globalisation and maritime labour in Norway after World War II".

18/14 May, **Tunç Durmaz,** "Energy Storage and Renewable Energy"

Norges Handelshøyskole

Norwegian School of Economics

NHH
Helleveien 30
NO-5045 Bergen
Norway

Tlf/Tel: +47 55 95 90 00
Faks/Fax: +47 55 95 91 00
nhh.postmottak@nhh.no
www.nhh.no