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A two-sector model of economic growth with endogenous technical change and pollution abatement

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Abstract We provide insights into the relationships between technological development, economic growth, and pollution accumulation, using a two-sector model of economic growth with endogenous technical change. In the model, output is produced using a polluting resource. Production can be used for either consumption or abatement of pollution. Scientists can be allocated between two research activities: resource-saving and abatement-augmenting technologies. Our results indicate conditional path dependency. Specifically, when the innovative capacity in the resource-saving research sector is sufficiently high, scientists are allocated to improve only the resource-saving technology, independently of the state of the technologies and environment. Consequently, the allocation of researchers is path-independent. When the innovative capacity in the abatement-augmenting research sector is sufficiently high, the optimal allocation of researchers depends on the initial level of the pollution stock or technologies but eventually will be directed to improve the abatement technology. We further characterize the optimal steady-state and off-steady-state dynamics and show that green growth is always socially optimal. By using a two-sector model, we address a lack of attention to multi-sector growth models in neoclassical growth theory and show that distinct results and transitional dynamics can emerge.

Keywords Green growth · Endogenous technical change · Path Dependency · Environmental Kuznets curve · Clean backstop · Climate change

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1 Introduction

Considering that they could power the machines of the Industrial Revolution and were rather easy to access, fossil fuels have been used extensively and increasingly to power the global economy ever since the Industrial Revolution. Although the widespread use of fossil fuels enabled high economic growth, and facilitated the creation of advanced and industrialized economies, they, either directly or indirectly, have given rise to significant pollution problems. Climate change, which refers to the phenomenon of increasing concentration of greenhouse gases (GHGs) in the world's atmosphere, and which is "the biggest market failure the world has seen" [68], is one example. Others include continued loss of biodiversity, exposure to air pollution and hazardous chemicals and their health-related impacts, and deterioration of freshwater sources [55].

It is evident that economic growth will be beneficial only if it does not undermine the ecological basis of our civilization. Achieving this, however, presents major challenges and will require significant changes in the composition of the global gross domestic product (GDP), and more emphasis on technological development [22]. A relevant question then is whether we can expect green growth, which means preserving and improving the environment while sustaining the growth of output and consumption.[1] If so, how can this be achieved, and what can be the role of technological progress?

To shed light on these questions and explore the link between environmental quality and economic growth, we use a two-sector model of economic growth with endogenous technical change. By using a two-sector model, we address a lack of attention to multi-sector growth models in neoclassical growth theory [67]. While the neoclassical growth theory is about the evolution of potential output, endogenous technical change is about the evolution of research while an estimated level of growth rate is attained in the long run. For example, Acemoglu et al [4] calibrate parameters constituting innovative capacities that enable an annual growth long-run rate of 2% per year [Table 2, 44]. We show that a two-sector model can lead to different results and transitional dynamics which can emerge depending on which sector R&D is directed at and how much resources are allocated to carry out research in each sector.

In this two-sector model, one sector produces a macro good, for which pollution (or emissions) is (are) an inevitable by-product. The other sector concentrates on pollution abatement. We incorporate resource-saving and abatement-augmenting technical changes in this model.[2] Moreover, we consider a stock pollutant dampened by a natural self-cleaning process. Hence, after accounting for the abatement efforts, if the amount of pollution is lower than the level of natural decay, the pollution stock will decrease, and equivalently, the environmental quality will improve.

In modeling technical change, we introduce an explicit research and development (R&D) sector, which uses scarce factors, that is, researchers, in producing new innovations [54, 2, 64, 4, 62]. R&D is treated like any other production activity that converts primary inputs (i.e., scientific labor and accumulated knowledge) into new knowledge [61, 7, 35]. As researchers are scarce, growth cannot be sustained by increasing the amount of these factors. In order to maintain economic growth, researchers need to become more productive over time, owing to spillovers from past research [1]. In this regard, the spillovers from previously accumulated knowledge make researchers more productive over time. Thus, researchers "stand on the shoulder of past giants" [3, p. 444].

We adopt a Ramseyan approach; there is a fictitious planning authority that maximizes a social intertemporal utility index, which is the sum of the discounted future stream of consumption and environmental amenities, with the discount rate being positive. We analyze both steady-state and off-steady-state dynamics when knowledge accumulation is endogenous. Our findings are mainly analytical characterizations of the optimal dynamics, which we complement with phase diagram analyses.

Our first result indicates that researchers are optimally allocated to the sector with sufficiently high innovative capacity. Innovative capacity is a composite measure that combines the probability of successful innovation and the relative increase in the accumulated knowledge upon successful innovation. Accordingly, all researchers are allocated to augment resource-saving technology when the innovative capacity in this sector is sufficiently high and the other way round. This mainly ensues from positing a trade-off between production and abatement, and thus, between resource-saving and abatement-augmenting technical change. To the best of our knowledge, we are not aware of any other studies that examine the dynamics of sustainable green growth using a bisectoral model of production and abatement where both sectors benefit from factor-augmenting technical change.

In contrast, the recent literature on endogenous technical change and the environment focuses solely on the trade-off between dirty and clean inputs (combined to produce the final good) and technologies [see 4, 49, 5, 6, 10].[3] Under this trade-off, an economy progressively directs its research toward the technology with higher accumulated technological knowledge. Thus, there is "path dependence" in the direction of technical change [6, 10].[4] Moreover, sustainable growth necessitates resources to be allocated progressively, from dirty to clean production and technology.

Nevertheless, due to our focus on the trade-off between resource-saving and abatement-augmenting technical change, our results indicate conditional path dependency. We use the term "conditional" because from an optimal perspective, the allocation of researchers depends on the level of the pollution stock and accumulated technological knowledge only when the innovative capacity in the abatement-augmenting (resource-saving) research sector is sufficiently high (low). In particular, when the innovative capacity in the resource-saving research sector is sufficiently high, resources for research are optimally allocated to augment

[1] Green growth has become a central objective of international organizations such as the Organization for Economic Cooperation and Development, the World Bank and the United Nations.

[2] Resource-saving technical change implies that the same level of output can be produced with less resource usage. On the other hand, abatement-augmenting technical change implies that the same level of abatement can be achieved with fewer use abatement inputs.

[3] Dirty technical change implies that the same level of production can be done with less use of the inputs, while still causing the same level of pollution. On the other hand, as clean production is pollution free, clean technical change implies the same level of clean production with less of the inputs.

[4] For the economic literature on endogenous technical change and environmental pollution (in particular, global warming and climate change), we refer the reader to Bovenberg and Smulders [20, 21], Goulder and Mathai [32], Popp [57, 58], Grimaud et al [34], Acemoglu et al [4, 6]. For the empirical literature on environment and endogenous technical change, the reader is referred to Newell et al [52], Popp [57], Acemoglu et al [6], Aghion et al [10]. For a conceptual review of economic growth and the environment, see Smulders et al [66].

only the resource-saving technology. Thus, independently of the severity of the pollution problem or how advanced the technologies are, researchers are allocated to work in the resource-saving research sector. As a result, the allocation of researchers is path-independent.

On the contrary, when the innovative capacity in the abatement-augmenting research sector is sufficiently high, the optimal allocation of researchers depends on the initial level of the pollution stock or technologies. As an example, for an initially low level of pollution, all researchers can initially be allocated to augment the resource-saving technology. A similar result can also ensue from low levels of both technologies. Thus, the allocation of researchers is conditional on the initial state of the environment and economy. Nonetheless, with the accumulating pollution problem and technical knowledge, all researchers will be allocated to augment only the abatement technology in the long run. Additionally, a research regime in which researchers are allocated to both research sectors simultaneously is only temporary.

Our second main result states that green growth is always socially optimal. By green growth, we mean that the economy settles on a balanced growth path (BGP), where the pollution stock converges to zero. Despite this monolithic result, time paths of the optimal solution trajectories can be different depending on the model's fundamentals. Once the pollution flow is fully abated eventually, the fossil fuels are entirely replaced by backstop technologies, which are clean and rely on everlasting resources [59].[5] This notion is related to the way we model abatement production, which can be interpreted as abatement of emissions (e.g., end-of-pipe technologies such as carbon capture and storage) or pollution-free renewable sources of energy (e.g., solar, wind, and hydropower) [30]. In this way, the economy settles on a green and BGP which can be sustained owing to technical progress and decreasing pollution stock.[6]

In addition to the analytical and theoretical discussions, we supplement our analysis with phase diagram analyses. In particular, we provide the conditions and illustrate the time profiles of economic variables that exhibit non-monotonous behaviors during transitions from pollution-intensive to pollution-free backstop production. Moreover, while the economy grows greener, we show that, depending on the initial conditions, the pollution stock either constantly declines or first increases before having to decrease. When the latter occurs, the pollution stock displays a pattern resembling an environmental Kuznets curve (EKC).

[5] Nordhaus [53] introduced the concept of backstop technology and analyzed the timing of the substitution of such technologies for fossil fuel resources.

[6] The phenomenon, which describes diminishing pollution and constant economic growth through technological progress, is also known as the Porter Hypothesis [60].

The relationship between pollution and economic growth allows us to relate our results to the EKC hypothesis and thus, to the corresponding literature on pollution and economic growth (see 69, 13, 27, 8, 23, 65).[7] EKC-related evidence for short-lived regional air and water pollutants is well known [see Table 1 in 48]. However, for pollutants with lower natural degradation rates, the existence of the EKC has been questioned [48, 65, 66] and assigned a weaker role to income in explaining the pollution dynamics. Hence, technological progress has recently been ascribed a more significant role in explaining the EKC behavior [65]. In this regard, our focus on endogenous technical change allows us to differentiate our study within this literature, and offer insights and new predictions as to the evolution of the environmental quality, and thus, the EKC-like behavior.

The remainder of the paper is structured as follows. Section 2 presents the model and the social planner problem. In Section 3, we characterize the optimum under endogenous technical change and investigate the economic and transitional dynamics. Section 4 summarizes and discusses the findings and presents a note on no technical progress. Section 5 concludes.

2 The model

We consider an economy wherein a composite macro good, $y(t)$, is produced from a polluting resource input denoted by $z(t)$, which is abundant and not restrained by any exhaustibility. The macro good production is done at each point in time according to the following technology:[8]

$$y(t) = A(t)z(t)^{\alpha}, \quad 0 < \alpha < 1. \tag{2.1}$$

$A(t)$ is the factor-augmenting term, and equivalently, the technological index, of the production technology that can be improved over time through R&D efforts. Factor-augmenting technical change, that is, increases in A, allows for output growth without increasing resource use, z.[9]

We acknowledge the fact that resources for production are limited in nature. Nonetheless, ongoing technical progress and discoveries of new fossil fuel reserves have shifted concerns away from the sustainability of growth

[7] The empirical literature on EKC starts with Grossman and Krueger [36, 37].

[8] In the rest of the paper, we use the terms macro good, output, production, income, and GDP interchangeably.

[9] In the context of energy production, improvements in fuel efficiency are an example of factor-augmenting technical change. One other example is material efficiency, a metric to express the degree of the use of a particular material required to produce particular a product [26]. Material efficiency can be improved by reducing the amount of the material that enters the production process and ends up in the waste stream without damaging the desired quality of the output [26, 42].

alongside resource scarcity to climate change and environmental degradation [40, 71, 46]. Moreover, as demonstrated by Golosov et al [31], if the resource is exhaustible, then a formulation of a polluting sector similar to ours cannot generate enough pollution to make it sufficiently costly. Furthermore, an abundant resource also allows us to put aside the complexities that would come with it and thus provide a tractable model.

In the economy, the macro good is used for consumption purposes, $c(t)$, and to finance environmental expenditures, $x(t)$. Consumption, environmental expenditures, and the macro good, accordingly, must satisfy the following feasibility condition:

$$y(t) \geq c(t) + x(t). \tag{2.2}$$

Pollution is a one-to-one by-product of resource use. Therefore, the flow of pollution also equals z. In the presence of environmental expenditures, it is possible to abate some fraction of the pollution. Let this abatement activity be denoted by $q(t)$. If we call z the gross level of pollution, then the net level of pollution can be denoted by

$$n(t) \equiv z(t) - q(t).$$

To simplify the analysis, we assume that the abatement activity is limited to the current flow of pollution. Therefore, $n(t) \geq 0$. In addition to abatement, the accumulation of pollution is dampened by a natural self-cleaning process, which, for simplicity is assumed to be proportional to the pollution stock. Let δ, the rate of natural decay of the pollution stock, be positive and constant. The law of motion of the pollution stock, $Z(t)$, is then given by

$$\dot{Z}(t) = n(t) - \delta Z(t), \;\; Z(0) = Z_0 > 0, \tag{2.3}$$

where the dot notation denotes change over time. Thus, for a variable, g, the change in its value in time is $\dot{g} \equiv dg/dt$. As Eq. (2.3) shows, if the net level of pollution is less than the level of natural decay, then the pollution stock will decrease, and equivalently, the environmental quality will improve. When the environment cannot regenerate itself (i.e., $\delta = 0$), environmental damage due to economic activity is irreversible. In Section 4, we look at the implications of irreversibility.

Pollution abatement requires the use of the macro good. Let $B(t)$ be the factor-augmenting term, that is, the technological index of the abatement technology. Then, abatement occurs as seen below:

$$q(t) = B(t)x(t)^{\beta}, \quad 0 < \beta < 1. \tag{2.4}$$

Similar to $A(t)$, the technological index of the abatement technology, $B(t)$, can be improved over time through R&D efforts.

Technology for knowledge generation—*à la Romer*—is:

$$\begin{aligned} &\dot{A}(t) = as_A(t)A(t), \;\; a > 0, \;\; A(0) = A_0 > 0 \;\text{ and} \\ &\dot{B}(t) = bs_B(t)B(t), \;\; b > 0, \;\; B(0) = B_0 > 0, \end{aligned} \tag{2.5}$$

where s_A and s_B are the amount of labor (researchers) who can improve the resource-saving technology (or resource efficiency) and abatement technology, respectively. Parameters a and b are sector-specific innovative capacities. Innovative capacity is a composite measure that combines the probability of successful innovation and relative increase in the accumulated knowledge consequent to successful innovation. If, for example, the amount of scientific labor allocated to augment the resource-saving technology is $s_A(t)$, then, for $t \in [t, t+dt]$, the relative increase in the accumulated knowledge upon successful innovation is $as_A(t)A(t)dt$ [9]. For a higher level of a (b), the resource-saving (abatement) R&D sector's innovative capacity increases.[10]

Both technologies benefit from the knowledge accumulated in the past, that is, there is state dependence in the innovation possibilities frontier. Accordingly, the spillovers from previously accumulated knowledge in one sector make researchers in that sector more productive over time. This is called the "standing on shoulders effect" [4] and can be seen as a natural feature in the current context. The reason is that improvements in the pollution-intensive technology (e.g., fossil fuel technology) may not directly translate into innovations in the non-polluting production (e.g., renewable energy technology). Therefore, we assumed away spillovers between the two research sectors.[11]

The economy includes a certain number of researchers whose population size is normalized to one; that is, $s_A + s_B \leq 1$. This is consistent with the existing literature on endogenous (directed) technical change and the environment [see 4, 62, 47, 33, 29] and the benevolent planner problem investigated by Hassler et al [39]. This formulation would be a close approximation to a more general setup as long as scientists constitute a small portion of the overall population, and the data reveals that

[10] The aggregate innovation production function, that is, Eq. (2.5), has constant returns to accumulated knowledge in the model. This is because if the returns to the accumulated knowledge are slightly higher, the model generates explosive growth. On the other hand, if there are decreasing returns to the accumulated knowledge, productivity growth gradually ceases.

[11] For specifications of two-sector growth models with knowledge spillovers, we refer the reader to [51].

the share of the researchers in R&D per million people is barely %0.1 [73, Table 5.13].[12]

Social welfare is a function of the level of consumption and pollution stock represented in a separable fashion. While higher consumption contributes to the social welfare, the pollution stock is damaging for the economy:

$$u\big(c(t), Z(t)\big) = \upsilon\left(c(t)\right) - h\left(Z(t)\right). \tag{2.6}$$

The utility from consuming the macro good is strictly increasing, $\upsilon' > 0$, strictly concave, $\upsilon'' < 0$, and satisfies $\lim_{c\to\infty} \upsilon'(c) = 0$ and $\lim_{c\to 0} \upsilon'(c) = +\infty$. The damage function, $h(Z)$, is strictly increasing, $h' > 0$, strictly convex, $h'' > 0$, and satisfies $h'(0) = 0$ and $\lim_{Z\to\infty} h'(Z) = +\infty$.

3 Optimal research and abatement regimes

3.1 Optimal program and first-order conditions

The objective of the planning authority is to maximize the discounted accumulated welfare. Let $\rho > 0$ be the constant rate of pure time preference. The optimal intertemporal allocation of the variables consists of time paths

$$\Big\{y(t), q(t), c(t), z(t), x(t), s_A(t), s_B(t), Z(t), A(t), B(t)\Big\}_{t=0}^{\infty},$$

which maximize the following social welfare problem

$$\begin{aligned} &\max \quad \int_0^\infty e^{-\rho t} u\big(c(t), Z(t)\big)dt \\ &\text{subject to} \quad y(t) = A(t)z(t)^\alpha, \\ &\qquad q(t) = B(t)x(t)^\beta, \\ &\qquad y(t) \geq c(t) + x(t), \\ &\qquad \dot{Z}(t) = n(t) - \delta Z(t) \\ &\qquad \dot{A}(t) = as_A(t)A(t), \\ &\qquad \dot{B}(t) = as_B(t)B(t), \end{aligned} \tag{3.1}$$

and the boundary and non-negativity constraints $c(t) \geq 0$, $x(t) \geq 0$, $y(t) \geq 0$, $z(t) \geq 0$, $q(t) \geq 0$, $z(t) \geq q(t)$, $s_A \geq 0$, $s_B \geq 0$, and $s_A + s_B \leq 1$.

Due to $\lim_{c\to 0} \upsilon'(c) = +\infty$, consumption is always strictly positive; that is, $c(t) > 0$ at all times. Thus, the macro good production, $y(t)$, and the input use, $z(t)$, are always strictly positive as well. Since the marginal product of the abatement input goes to infinity when $x(t)$ approaches zero, abatement expenditure is always strictly positive; $x(t) > 0$. Hence, in any optimum, the economy always abates some fraction of the gross pollution, that is, $q(t) > 0$ and $q(t) \leq z(t)$. The society optimally splits the output between consumption and abatement input. Therefore, the feasibility constraint always holds, $y(t) = c(t) + x(t)$. From here on, we drop the time index, t, unless it causes confusion.

[12] This assumption also allows us to avoid any scale effect on output growth [43, 38]. For example, if the number of researchers was subject to exponential growth, the growth rate of the output in our model would itself grow exponentially.

The current-value Lagrangian that corresponds to the social maximization problem given by Eq. (3.1) reads as

$$\begin{aligned} \mathcal{L}(\cdot) = \upsilon\left(c\right) - h\left(Z\right) + p\left(Az^\alpha - c - x\right) \\ - \lambda\left(z - Bx^\beta - \delta Z\right) + \gamma\left(z - Bx^\beta\right) + \lambda_A a s_A A \\ + \lambda_B b s_B B + \omega\left(1 - s_A - s_B\right) + \nu_A s_A + \nu_B s_B. \end{aligned} \tag{3.2}$$

For reasons explained earlier, we discard the positivity constraints for c, y, q, x, and z. Since the linear specification of the R&D process can induce corner paths for research, that is, $s_A = 0$ or $s_B = 0$, we keep the corresponding non-negativity constraints. In Eq. (3.2), while p is the Lagrange multiplier associated with the feasibility constraint and reflects the social value of the macro good, λ is the social cost of pollution. In order to have λ positive, we have put minus sign in front of it in the Lagrangian. Moreover, γ is the multiplier associated with full abatement (FA; i.e., $n = 0$). Finally, λ_A and λ_B are the social values of factor efficiency improvements for the macro good and abatement, respectively. Let $\mu \equiv \lambda - \gamma$ denote the net opportunity cost of the pollution when it is fully abated; that is, $\gamma \geq 0$.

The necessary and sufficient conditions to maximize the current value Lagrangian with respect to c, z, and x yield

$$\upsilon'(c) = p, \tag{3.3}$$
$$\alpha p A z^{\alpha-1} = \mu, \tag{3.4}$$
$$p = \beta B x^{\beta-1}\mu, \tag{3.5}$$
$$\gamma\left(z - Bx^\beta\right) = 0,\ \gamma \geq 0,\ z - q \geq 0. \tag{3.6}$$

While Eq. (3.3) equates the cost of consumption to its marginal utility, Eq. (3.4) does the same for the marginal product of the polluting resource and its only cost: the environmental externality. Eq. (3.5) yields that the marginal product of environmental expenditure is equal to its marginal cost. From Eq. (3.6), $\gamma = 0$ when capturing takes place partially (i.e., $n > 0$). In this case, the net and gross cost of pollution are equal, $\mu = \lambda$. When pollution is fully abated (i.e., $n = 0$), $\gamma \geq 0$ and $\mu = \lambda - \gamma$.

The conditions for optimal allocation of researchers verify

$$a\lambda_A A \le \omega, \text{ with equality if } s_A > 0 \text{ and} \quad (3.7)$$
$$b\lambda_B B \le \omega, \text{ with equality if } s_B > 0, \quad (3.8)$$

where ω denotes the (social) wage of a researcher. When researchers are allocated to both sectors, then the marginal social values created by doing research in both research sectors are equal: $a\lambda_A A = B\lambda_B B = \omega$. Otherwise, the social value of doing research in one of these sectors is higher if there is a corner solution; for example, $a\lambda_A A = \omega > b\lambda_B B$ if $s_A = 1$.

The usual no-arbitrage conditions are given by the following:

$$\frac{\partial \mathcal{L}}{\partial Z} = -\rho\lambda + \dot{\lambda} : \dot{\lambda} = (\rho + \delta)\,\lambda - h'(Z), \quad (3.9)$$
$$\frac{\partial \mathcal{L}}{\partial A} = \rho\lambda_A - \dot{\lambda}_A : \dot{\lambda}_A = (\rho - as_A)\lambda_A - pz^{\alpha}, \quad (3.10)$$
$$\frac{\partial \mathcal{L}}{\partial B} = \rho\lambda_B - \dot{\lambda}_B : \dot{\lambda}_B = (\rho - bs_B)\lambda_B - \mu x^{\beta}. \quad (3.11)$$

Eq. (3.9) describes the necessary condition for the dynamics of the pollution stock on an optimal solution path. Thus, it represents the optimal dynamics of λ. Eqs. (3.10) and (3.11) are the necessary conditions for the accumulation of the technological indexes when researchers are optimally allocated to the research sectors.

Lastly, the following transversality condition has to be verified:

$$\lim_{t\to\infty} e^{-\rho t}\left[-\lambda(t)Z(t) + \lambda_A(t)A(t) + \lambda_B(t)B(t)\right] = 0 \quad (3.12)$$

3.2 Preliminaries

We establish some preliminary results and properties that we will employ later in the analyses. Let $y_z \equiv \partial y/\partial z$ denote the marginal product of output with respect to the resource use. Similarly, we denote the marginal product of abatement with respect to the environmental expenditure by $q_x \equiv \partial q/\partial x$. Then, dividing Eq. (3.4) by Eq. (3.5) yields

$$y_z = {q_x}^{-1}, \quad (3.13)$$

which is a static efficiency condition. Eq. (3.13) shows the equalization of the marginal rates of transformation between the output and pollution along any efficient path at each point in time. Rearranging Eq. (3.13) gives

$$\sigma \equiv \frac{x}{y} = \alpha\beta\frac{q}{z}, \quad (3.14)$$

where σ is the share of environmental expenditure in the GDP. Notice that, first, the FA constraint (i.e., $q \le z$) defines an upper limit for the amount of resources that can be spent on abatement. Second, the fact that there is always some abatement activity, $q > 0$, allows us to write $0 < \sigma \le \alpha\beta$. As $\alpha < 1$ and $\beta < 1$, the share of abatement expenditure in total output is less than one; that is, $\sigma < 1$. In other words, it is not possible to use the whole output for environmental expenditure.

For $\epsilon > 0$ and $\nu > 0$, the instantaneous welfare of the society is represented by the following functional form:

$$u(c, Z) = \frac{1}{1-\epsilon}c^{1-\epsilon} - \frac{1}{1+\nu}Z^{1+\nu}, \quad (3.15)$$

where $1/\epsilon$ and $1/\nu$ is the consumption and Frisch elasticity of intertemporal substitution, respectively. As an alternative interpretation, $\nu > 0$ allows us to work with a strictly convex damage function, whose certain appealing features are discussed by Moreaux and Withagen [50].[13]

Considering the explicit isoelastic separable preferences, the net level of pollution and its opportunity cost can be expressed as functions of A, B, and σ. In particular, the static efficiency condition, Eq. (3.14), the optimality conditions, Eqs. (3.3), (3.4), and (3.5), and the functional form given by Eq. (3.15) imply

$$n = N(A, B)\,(\alpha\beta - \sigma)\,\sigma^{-\frac{1-\beta}{1-\alpha\beta}}, \quad (3.16)$$
$$\mu = M(A, B)(1-\sigma)^{-\epsilon}\sigma^{\frac{(1-\beta)(1-\alpha+\alpha\epsilon)}{1-\alpha\beta}}, \quad (3.17)$$

where N and M are constants that depend on the parameters of the technology. To be precise, $N(A, B) = (\alpha\beta)^{\frac{\alpha\beta}{1-\alpha\beta}}(A^{\beta}B)^{\frac{1}{1-\alpha\beta}}$ and $M(A, B) = \alpha(\alpha\beta)^{-\frac{1-\alpha+\alpha\epsilon}{1-\alpha\beta}}\left(A^{1-\beta-\epsilon}B^{-(1-\alpha+\alpha\epsilon)}\right)^{\frac{1}{1-\alpha\beta}}$.

In the following, we will explore the relationship between the share of abatement expenditure in the GDP, σ, and net level of pollution, n, expressed by Eq. (3.16). Let $\varphi_Z(\sigma)$ denote the product of the last two terms in Eq. (3.16):

$$\varphi_Z(\sigma) \equiv (\alpha\beta - \sigma)\sigma^{-\frac{1-\beta}{1-\alpha\beta}}. \quad (3.18)$$

It is immediately apparent that $\varphi_Z(0) = \infty$, $\varphi_Z(\alpha\beta) = 0$ and $d\varphi_Z/d\sigma < 0$. Thus, $n(\sigma)$ is a strictly decreasing function of σ. Accordingly, when it is optimal to use a higher share of the output toward the abatement activity, the net level of pollution diminishes. In contrast, for a lower share of abatement expenditure in the GDP, and

[13] Quite a few other studies have followed this mainstream approach, where damage adversely affects the social welfare (e.g., 72, 41, 56, 18, 50). Alternatively, Chakravorty et al [24, 25], Amigues and Moreaux [11], Amigues et al [12], Gerlagh et al [30] and Kollenbach [45] set a ceiling on the accumulated pollution stock. The incentive behind choosing a ceiling is that "it represents a threshold beyond which a catastrophe takes place" [50].

therefore, lower abatement activity, the net level of pollution will be higher.

A similar analysis can be conducted by invoking the formula for μ given by Eq. (3.17). Let the product of the last two terms be denoted by

$$\varphi_\lambda(\sigma) \equiv (1-\sigma)^{-\epsilon}\sigma^{\frac{(1-\beta)(1-\alpha+\alpha\epsilon)}{1-\alpha\beta}}. \tag{3.19}$$

Consequently, it follows that $d\varphi_\lambda/d\sigma > 0$, which implies

$$d\mu/d\sigma > 0. \tag{3.20}$$

That is, there is a positive relation between the net opportunity cost of pollution and σ. Moreover, as the share of abatement expenditure, σ, lies inside the interval $(0, \alpha\beta]$, we can deduce the following limiting properties:

$$\varphi_\lambda(0) = 0 \quad \text{and} \quad \bar{\varphi}_\lambda \equiv \varphi_\lambda(\alpha\beta) > 0,$$

where $\bar{\varphi}_\lambda$ denotes the upper bound of φ_λ. When the economy performs PA, i.e., $q < z$, the corresponding co-state variable satisfies $\gamma = 0$. Therefore, the net opportunity cost of pollution is equal to the social cost of pollution, $\mu = \lambda$. As a consequence, $d\varphi_\lambda/d\sigma > 0$ implies that the social cost of pollution increases with σ. Thus, there is a monotonic relationship between σ and λ due to Eq. (3.20), which implies that σ can be deemed a function of λ.

In the following, we will present our first major outcome, which indicates, from a social planner's perspective, that FA is inevitable in the long run.

Theorem 1 *The share of abatement expenditure, σ, converges to its maximum value of $\alpha\beta$ over time for any solution.*

Proof. The proof is provided in Appendix A. □

Despite this monolithic result, we can categorize the optimal solutions into two abatement regimes. In one regime, the pollution flow is fully abated in finite time. In the other one, FA is achieved asymptotically. To simplify the analysis and be concise, we only focus on abatement dynamics that lead to FA and, with it, a balanced and pollution-free (green) economic growth in finite time. As will be clear later on, FA is inevitable in finite time when we do not consider pollution decay. This assumption was made by Tahvonen [70] and Rouge [62] to make the analysis simpler. We relax the no-pollution-decay assumption as long as FA is attained in finite time.

We first analyze optimal research and abatement regime, and economic dynamics under the proviso that resource-saving research is sufficiently innovative. We then investigate the research and economic dynamics when abatement-augmenting research regime is sufficiently innovative.

3.3 Sufficiently innovative resource-saving research

3.3.1 Optimal research regime

The sectoral innovative capacities (a and b) play major roles when optimally allocating the researchers to the two R&D sectors. In particular, when the productivity of research in resource-saving R&D is higher than that in abatement R&D weighted by the output elasticity of the resource, α, it is always optimal to only augment the resource-saving technology.

Proposition 1 *If the innovative capacity in the resource-saving research sector is sufficiently high, that is, if $a > \alpha b$, it is always optimal to allocate all scientists to augment the resource-saving technology. Thus, $s_A(t) = 1$ for all t at any solution.*

Proof. The proof is provided in Appendix B. □

Consequently, under the parametric condition allowing for a sufficiently high innovative capacity in the resource-saving R&D sector, technical change is exogenous in the sense that it is unaffected by the decision variables. Despite this monolithic result, there are different optimal dynamics through which the economy converges to a balanced growth path. In the following, we investigate the economic dynamics when it is optimal to allocate resources to augment the resource-saving technology.

When $s_A(t) = 1$ for all t at any solution, the technological index for production efficiency grows at the constant rate of $\dot{A}/A = a$.

3.3.2 Permanent FA regime

We first focus on the case where pollution flow is always fully abated. The necessary and sufficient condition for FA at time $\bar{t}$ is that the net opportunity cost of pollution is less than the social cost of pollution; that is, $\bar{\mu}(\bar{t}) \leq \lambda(\bar{t})$, where $\bar{\mu}(\bar{t}) \equiv M(A(\bar{t}), B_0)\bar{\varphi}_\lambda$.[14] Moreover, a sufficient condition for $\bar{\mu}(t) \leq \lambda(t)$ to hold over $t \in [\bar{t}, \infty)$ is that the growth rate of the social cost of pollution remains higher than that of the net opportunity cost of pollution. This can be stated formally as $\theta_{\bar{\mu}} < \theta_\lambda$ for $t \geq \bar{t}$, where $\theta_f \equiv \dot{f}(t)/f(t)$ denotes the growth rate of any time

[14] Notice that under a resource-saving research regime, all scientists are allocated to the resource-saving research sector. Therefore, $B(t) = B_0$.

function, $f(t)$. In open form, the inequality $\theta_{\bar{\mu}} < \theta_\lambda$ can be restated as a simple and transparent inequality. This leads us to the following proposition.

Proposition 2 *When it is optimal to constantly allocate all the scientists to augment the resource-saving technology, that is, $s_A(t) = 1$ for all t at any solution, the growth rate of the social cost of pollution is greater than the growth rate of the net opportunity cost of pollution, $\theta_\lambda > \theta_{\bar{\mu}}$, from $\bar{t}$ onwards if and only if $-\delta\nu > a(1-\beta-\epsilon)/(1-\alpha\beta)$.*

Proof. The proof is provided in Appendix C. □

Notice that $1-\beta < \epsilon$ is a necessary condition for the growth rate of the net opportunity cost of pollution to be smaller than the growth rate of the social cost of pollution from $\bar{t}$ onwards. Incidentally, there is a large body of empirical literature that attempts to estimate ϵ, or, equivalently, the inverse of the elasticity of intertemporal substitution. In particular, the evidence suggests that $\epsilon > 1$ [14, 17, 19, 16, 15, 63, 28]. From an empirical standpoint, an important necessary condition is satisfied for $\theta_{\bar{\mu}} < \theta_\lambda$ for $t \geq \bar{t}$ to hold.

One may ask how the net opportunity cost of pollution at time $\bar{t}$ can be smaller than the social cost of pollution. First, notice that at time $t = \bar{t}$, the cost of pollution to the society is (cf. Eq. C.2 in the appendix)

$$\lambda(\bar{t}) \equiv \frac{Z(\bar{t})^\nu}{\rho + \delta(1+\nu)}.$$

It is clear that $\lambda(\bar{t})$ is increasing and unbounded in $Z(\bar{t})$. This suggests that a sufficiently large level of pollution stock, $Z(\bar{t})$, gives ample motivation for FA. If the environment gets severely polluted, then the social planner will naturally find it beneficial to surmount this problem by allocating resources for FA. Furthermore, $\bar{\mu}$ is a decreasing function of $A(t)$. Hence, in a similar vein, FA is optimal for a sufficiently high level of $A(\bar{t})$. In this case, the motivation for FA stems from technological efficiency: sufficiently high productivity substitutes the polluting resource in production, and in turn, decreases the social cost of FA. A sufficiently high B_0 will also allow for FA from $t = \bar{t}$ onward as long as $\theta_{\bar{\mu}} < \theta_\lambda$ for $t \geq \bar{t}$. Lastly, if the net opportunity cost of pollution is smaller than the social cost of pollution for $\bar{t} = 0$, FA is the optimal scenario from the beginning provided that $-\delta\nu > \bar{\theta}^A_\lambda$.

Balanced growth path The dynamics of the variables under a perpetual FA regime in which $\bar{\mu}(0) \leq \lambda(0)$ and $\theta_{\bar{\mu}} < \theta_\lambda$ are fairly straightforward. (We call an optimal abatement regime with FA from the start, i.e., from $\bar{t} = 0$, as a perpetual FA regime). First, since $\sigma \leq \alpha\beta$, the share of the abatement expenditure in the GDP will attain its maximum value: $\sigma = \alpha\beta$. Thus, the pollution stock will contract at the rate of natural decay: $\theta_Z = -\delta$. Further, given that $\theta_A \equiv \dot{A}/A = a$, one can calculate the growth rates for the variables from Eqs. (3.27) and (3.28). Consequently, the common growth rate for output, abatement expenditure, and consumption is $a/(1-\alpha\beta)$. Similar calculations show that the growth rate for both resource use and abatement activity is $\beta a/(1-\alpha\beta)$.

Pollution-free backstop technology In the presence of abundant resource, the abatement activity in our model can be interpreted as a backstop technology, where a cheaper backstop leads to lower pollution [50]. In this regard, a perpetual FA regime implies that the pollution-free production fully substitutes pollution-intensive production; that is, the economy relies only on pollution-free backstop technologies.

3.3.3 Before FA

It is evident that once it is costlier on the margin to fully abate the pollution from the start, that is, $\bar{\mu}(0) > \lambda(0)$, it will be optimal to pursue PA. This suggests that either the initial pollution stock is not sufficiently large or that the starting level of resource efficiency is not sufficiently high (or both). For the prior, the social planner will not find it beneficial to surmount the pollution problem by allocating resources for FA. For the latter, FA will not be optimal as the technological efficiency will not be sufficiently high to substitute the polluting resource in production. If, however, the social cost of pollution corresponding to an FA regime grows faster than the net opportunity cost of pollution, partial abatement (PA) will be pursued for a temporary period. In other words, if $-\delta\nu > \bar{\theta}^A_\lambda$ holds, the marginal cost of FA will eventually become lower than the social cost of pollution. It will then be optimal for the economy to abate the pollution fully. This leads us to the following proposition.

Proposition 3 *FA is optimal from $\bar{t}$ onward if the initial net opportunity cost of pollution is bigger than the social cost of pollution, that is, $\bar{\mu}(0) > \lambda(0)$, and $-\delta\nu > a(1-\beta-\epsilon)/(1-\alpha\beta)$.*

Proof. The proof is provided in Appendix D. □

Because the timing of the switch to an FA regime crucially depends on the growth rate of the social cost of pollution in an FA regime and the growth rate of the net opportunity cost of pollution, a few comments are in order. Notice that when the economic activity, and thus, pollution, has an irreversible impact on the environment (i.e., $\delta = 0$), the left-hand side of the inequality provided in Proposition 3 becomes zero. As the rate of natural decay is zero, the growth rate of the social cost of pollution is always higher than the growth rate of the marginal cost of FA. A zero natural decay rate indicates an accelerated switch to an FA regime. Considering, for example, long-lived GHGs (e.g., CO_2) with very low rates of natural decay, it can be beneficial for the society to fully abate the emissions sooner. If not, future generations can inherit a severe pollution problem, and in turn, be obliged to decrease their consumption levels substantially.

Considering the impact of the elasticity of the intertemporal substitution parameter, that is, $1/\epsilon$, a low elasticity of intertemporal substitution decreases the preferability of substituting current consumption for future consumption. Thus, high consumption, causing high pollution today, and low consumption in the future to decrease the pollution stock, become less beneficial for the society. Accordingly, for a low elasticity of intertemporal substitution, it is more favorable to purse FA sooner. The intuition for the case in which elasticity of the intertemporal substitution is high can be interpreted similarly.

A lower (higher) level of ν is equivalent to a higher (lower) Frisch elasticity of intertemporal substitution. In this case, it becomes optimal to pursue FA at a sooner (later) point in time because for a high level of the Frisch elasticity of intertemporal substitution (i.e., for a low ν), the preferability of substituting the environment today for that in the future increases. Therefore, it becomes more beneficial for the society to start FA earlier and have lower pollution in the future. A similar explanation can be made when ν is high.

Lastly, consider the parameters α and β. When β is high, the return to the abatement input is high, allowing for lower use of the abatement input for the same level of abatement activity. Thus, the marginal cost of FA decreases. On the other hand, when α is high, the marginal productivity of the resource is high. Accordingly, the use of the resource is low for the same level of production. Less use of the resource, and thus, lower pollution, reduces the marginal cost of FA. As a result, a high β or α leads to the FA regime being pursued earlier.

3.3.4 Transition dynamics

Transition dynamics of Z and λ When it is costlier to fully abate pollution initially, the economy can exhibit diverse dynamics during the temporary PA phase. We need to go through a few calculations to demonstrate this.

Using Eqs. (2.3), (3.9), and (3.15), we start by writing the dynamical system that describes the law of motion of the growth rates of the social cost of pollution and pollution stock over time:

$$\begin{aligned}\theta_\lambda &= \rho + \delta - \frac{Z^\nu}{\lambda},\\ \theta_Z &= \frac{n}{Z} - \delta.\end{aligned} \tag{3.21}$$

Further, to characterize the optimal trajectory in the $(\theta_Z, \theta_\lambda)$ plane, we, first, time differentiate the latter expression:

$$\begin{aligned}\frac{\dot{\theta}_Z}{\theta_Z + \delta} &= \theta_n - \theta_Z,\\ &= \bar{\theta}_Z^A - K_Z(\sigma)\theta_\sigma - \theta_Z,\end{aligned} \tag{3.22}$$

where $\bar{\theta}_Z^A \equiv \frac{\beta a}{1-\alpha\beta}$ and $K_Z(\sigma) \equiv \frac{\beta[(1-\alpha)\sigma+\alpha(1-\beta)]}{(1-\alpha\beta)(\alpha\beta-\sigma)} > 0$, and the growth rate of the net pollution, θ_n, is calculated from Eq. (3.16). Moreover, from Eq. (3.17), differentiating the social cost of pollution for a PA phase yields

$$\theta_\lambda = \bar{\theta}_\lambda^A + K_\lambda(\sigma)\theta_\sigma, \tag{3.23}$$

where $\bar{\theta}_\lambda^A \equiv a(1-\beta-\epsilon)/(1-\alpha\beta)$ and $K_\lambda(\sigma)$ is equivalent to

$$\frac{\epsilon[(1-\alpha)\sigma + \alpha(1-\beta)] + (1-\alpha)(1-\beta)(1-\sigma)}{(1-\alpha\beta)(1-\sigma)} > 0.$$

Given that $\bar{\theta}_\lambda^A$ is a constant, Eq. (3.23) indicates that there is a monotonic relationship between the growth rates of the social cost of pollution, θ_λ, and the share of the abatement expenditure in the output, θ_σ. Notice that because $\theta_\lambda \geq -\nu\delta > \bar{\theta}_\lambda^A$, the share of expenditure in the economy grows in time until it is optimal to fully abate the pollution flow.

Let $\kappa(\sigma) \equiv K_\lambda(\sigma)/K_Z(\sigma)$, which is a decreasing function of the share of abatement expenditure in the GDP. It varies between $1-\alpha+\alpha\epsilon > 0$ and 0 as σ varies between 0 and $\alpha\beta$. Using Eq. (3.23), if one substitutes for θ_σ in Eq. (3.22), the change in the growth rate of the pollution stock can finally be shown as

$$\kappa(\sigma)\frac{\dot{\theta}_Z}{\theta_Z + \delta} = [\bar{\theta}_\lambda^A + \kappa(\sigma)\bar{\theta}_Z^A] - \theta_\lambda - \kappa(\sigma)\theta_Z. \tag{3.24}$$

On the other hand, time differentiating the growth rate of the social cost of pollution, θ_λ, given by Eq. (3.21), yields

$$\frac{\dot{\theta}_\lambda}{\rho + \delta - \theta_\lambda} = \theta_\lambda - \nu\theta_Z. \tag{3.25}$$

We can now characterize the optimal trajectory in the $(\theta_Z, \theta_\lambda)$ plane. While the isocline $\dot{\theta}_\lambda = 0$ defines a linear line, $\theta_\lambda = \nu\theta_Z$, in the phase plane, the isocline $\dot{\theta}_Z = 0$ defines a family of lines that are parameterized by σ,

$$\theta_\lambda = [\bar{\theta}_\lambda^A + \kappa(\sigma)\bar{\theta}_Z^A] - \kappa(\sigma)\theta_Z. \tag{3.26}$$

A close look at Eq. (3.26) reveals that all the lines cross at the point $(\bar{\theta}_Z^A, \bar{\theta}_\lambda^A)$ in the phase plane.

Figure 1 demonstrates the phase plane for $(\theta_Z, \theta_\lambda)$ by plotting the isoclines $\dot{\theta}_Z = 0$ and $\dot{\theta}_\lambda = 0$. First, notice that the FA constraint requires $\theta_Z \geq -\delta$. Moreover, the growth rate of the social cost of pollution cannot exceed the sum of the time preference and rate of natural decay: $\theta_\lambda \leq \rho + \delta$. Otherwise, one can see from Eq. (3.21) that the pollution stock becomes negative for a positive λ. Consequently, the relevant $(\theta_Z, \theta_\lambda)$ pairs are located to the right of the vertical line $\theta_Z = -\delta$ and below the horizontal line $\theta_\lambda = \rho + \delta$. Moreover, the transversality condition requires that $e^{-\rho t}\lambda(t)Z(t)$ cannot increase when t is sufficiently high. Therefore, $\theta_\lambda(t) + \theta_Z(t) - \rho < 0$ when t is large, and all trajectories above line indicated by T cannot be optimal.

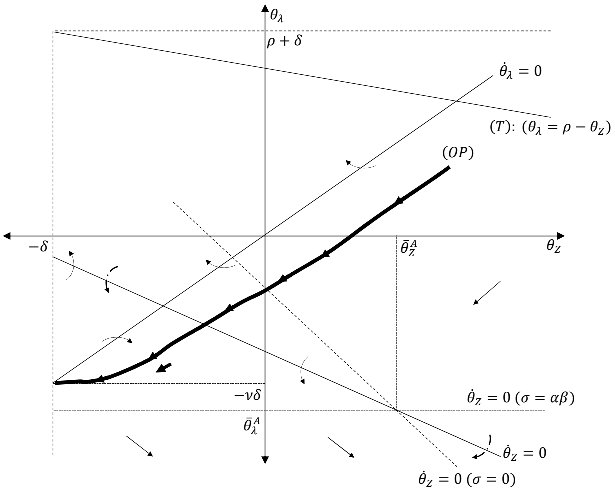

Fig. 1: *Phase diagram analysis. Notice that the share of abatement (σ) grows in time since $\theta_\lambda > \bar{\theta}_\lambda^A$. With the growing share of abatement, the isocline $\dot{\theta}_Z = 0$ rotates counterclockwise.*

From Eq. (3.23), the growth rate of σ is positive whenever $\theta_\lambda > \bar{\theta}_\lambda^A$. In other words, when the $(\theta_Z, \theta_\lambda)$ pairs are located above the horizontal line denoting $\bar{\theta}_\lambda^A$, the share of the abatement expenditure in the total output grows in time. Because the slope of the isocline $\dot{\theta}_Z = 0$ given in Eq. (3.24) varies between $1 - \alpha + \alpha\epsilon > 0$ and 0 as σ varies between 0 and $\alpha\beta$, it rotates counterclockwise and converges to the horizontal line $\bar{\theta}_\lambda^A$ owing to the rising share of the abatement expenditure in time (cf. Eq.(3.23)). As a result, the optimal trajectory (OP in Figure 1) moves toward $(-\delta, -\nu\delta)$. As $-\delta\nu > \bar{\theta}_\lambda^A$, the $(-\delta, -\nu\delta)$ pair, which corresponds to the growth rates of Z and λ in an FA regime, respectively, is attained in finite time, that is, when $t = \bar{t}$.

Transition dynamics of the economic variables The fact that the growth rates of the social cost of pollution and the pollution stock, θ_λ and θ_Z, can take positive values during a PA phase can cause the dynamics of y, z, and c to be non-monotonous. In exploring the time paths for these variables, we express them as functions of A, B, and σ (cf. Eqs. (3.16) and (3.17)). To be concise, we only present the expressions for resource use and abatement expenditure. Note that y and q can be calculated from Eqs. (2.1) and (2.4), respectively.

$$z = (\alpha\beta)^{\frac{1}{1-\alpha\beta}} \left(A^\beta B\right)^{\frac{1}{1-\alpha\beta}} \sigma^{-\frac{1-\beta}{1-\alpha\beta}}, \tag{3.27}$$

$$x = (\alpha\beta)^{\frac{\alpha}{1-\alpha\beta}} \left(AB^\alpha\right)^{\frac{1}{1-\alpha\beta}} \sigma^{\frac{1-\alpha}{1-\alpha\beta}}. \tag{3.28}$$

Consider the polluting resource, z. From Eqs. (3.27) and (3.23) one can show that for a sufficiently high growth rate in the social cost of pollution, the use of the polluting resource will diminish initially:

$$\begin{aligned} &\theta_z < 0 \quad \text{if} \quad \theta_\lambda > \frac{\beta a}{1-\beta} K_\lambda(\sigma) + \bar{\theta}_\lambda^A, \\ &\theta_z \geq 0 \quad \text{otherwise.} \end{aligned} \tag{3.29}$$

The growth dynamics for the macro good, y, and consumption, c, can be derived similarly.

Remark 1 Prior to $\bar{t}$, which corresponds to a PA phase, the output, resource use, and consumption can exhibit non-monotonous behaviors. Further, the pollution stock either constantly declines or first increases before decreasing.

We can verify that both the abatement expenditure and abatement activity, x and q, respectively, increase during the two time phases. This reflects the fact that when the pollution stock rises and the environmental quality deteriorates, there will be an increasing concern for environmental pollution, which will lead to increasing abatement expenditure and activity.

Environmental Kuznets Curve Depending on the initial conditions, there may be an initial phase in which the pollution stock increases; that is, $\theta_Z > 0$. In Figure 1, this corresponds to OP being located to the right of the θ_λ axis. This leads us to make the following remark.

Remark 2 When the optimal trajectory, OP, passes through the positive θ_Z axis, the pollution stock displays a pattern resembling an EKC.

EKC describes the evolution of a stock pollutant in time. During an initial time phase, the pollution stock increases and causes the environmental quality to deteriorate. In the second time phase, the pollution stock shrinks over time. The basic idea is to link the EKC-like behavior to the economic growth. Accordingly, in the early stages of economic development, the environmental quality can deteriorate. However, with increasing income, the environmental quality improves again.

The following remark summarizes the main findings thus far:

Remark 3 When the innovative capacity in the resource-saving R&D sector is sufficiently high,

- All resources for research are directed at improving the resource-saving technology from the start.
- If the marginal cost of fully abating the pollution is smaller than the social cost of pollution initially, the economy should permanently purse FA. The economic output, resource use, consumption, and abatement all increase at constant rates over time. The economy settles on a BGP right from the start.
- If the marginal cost of fully abating the pollution is bigger than the social cost of pollution initially, the optimal path is composed of two successive time phases. During a first time phase, the economy performs PA. While the abatement efforts continuously rise during this time phase, the output, resource use, and consumption may experience non-monotonous evolutions. The pollution stock size either continually declines, or either first increases before having to decrease. In finite time, the economy enters the permanent FA regime and settles on a BGP.

3.4 Sufficiently innovative abatement-augmenting research

In our pursuit of the optimal research policy, we continue our analysis by postulating a sufficiently high innovative capacity in the abatement-augmenting R&D; that is, $b > a/\alpha$. By doing this, we exhaust the set of possible parameters. Notice that when $b = a/\alpha$, it is optimal to direct all researchers to augment the resource-saving technology before FA is pursued permanently. For a permanent FA regime, however, the social wages in both research sectors are equal. Therefore, in this knife-edge scenario, the social planner is indifferent between the two research regimes (see Eq. B.3).

The previous analysis demonstrated that for a sufficiently high innovative capacity in the resource-saving research sector, all researchers were optimally allocated to augment the resource-saving technology. In this section, we show that for a sufficiently high innovative capacity in the abatement-augmenting research sector, all resources for research will eventually be directed to augment the abatement technology. Nevertheless, the optimal research regime can be different in the early stages of the economy.

3.4.1 Permanent FA regime

To begin with, suppose that the social cost of pollution becomes greater than the net opportunity cost of pollution at time $\bar{t}$ and FA is pursued from then onwards. Therefore, $\bar{\mu}(t) \leq \lambda(t)$ for $t \geq \bar{t}$. The inequality provided by Proposition 2 ensures that FA is pursed permanently. This leads us to the following corollary:

Corollary 1 *When the innovative capacity in the abatement-augmenting research sector is sufficiently high, that is, $\alpha b > a$, the growth rate of the net opportunity cost of pollution is smaller than the growth rate of the social cost of pollution from $\bar{t}$ onwards if $-\delta\nu > \bar{\theta}_\lambda^A$.*

Proof. The proof is provided in Appendix E. □

Optimal research regime When it is optimal to fully abate the pollution for $t \geq \bar{t}$, it is fairly straightforward to determine the optimal research regime. This is stated more formally in the following corollary:

Corollary 2 *For a sufficiently high innovative capacity in the abatement-augmenting research sector ($\alpha b > a$), all scientists are allocated to improve the abatement-augmenting technology from $\bar{t}$ onward, that is, $s_B(t) = 1$ for $t \in [\bar{t}, \infty)$ if $-\delta\nu > \bar{\theta}_\lambda^A$. Furthermore, the pollution flow is fully abated, and all scientists are directed at the abatement-augmenting research sector from $t = 0$ onward, that is, $\bar{t} = 0$, if $\bar{\mu}(0) \leq \lambda(0)$.*

Proof. The proof is provided in Appendix F. □

Corollary 2 indicates that if the environment is severely polluted initially, there is considerable incentive for the social planner to allocate the resources toward FA and abatement-augmenting research when the sectoral innovative capacity in the latter is sufficiently high. Furthermore, FA and abatement-augmenting research are optimal for a sufficiently high level of accumulated knowledge in both research sectors. In this case, the motivation for FA and allocating all researchers to do abatement-augmenting research stems from technological efficiency.

3.4.2 Before FA

It is optimal for the economy to fully abate its pollution flow later when it is costlier to do so initially; that is, $\bar{\mu}(0) \geq \lambda(0)$. An analysis similar to the one conducted for Proposition 3 results in the following corollary:

Corollary 3 *If the net opportunity cost of pollution is bigger than the social cost of pollution initially, that is, $\bar{\mu}(0) > \lambda(0)$, but $-\delta\nu > \bar{\theta}_\lambda^A$, then FA is pursued permanently for $t \geq \bar{t}$.*

Proof. The proof is provided in Appendix G. □

Considering that research can take place in both sectors, the growth rate of the social cost of pollution during a PA phase can be calculated from Eq. (3.17):

$$\theta_\lambda = \bar{\theta}_\lambda^A s_A + \bar{\theta}_\lambda^B s_B + K_\lambda(\sigma)\theta_\sigma, \tag{3.30}$$

where $\bar{\theta}^B_\lambda \equiv -b(1-\alpha+\alpha\epsilon)/(1-\alpha\beta)$. Since the minimum growth rate of the social cost of pollution is $-\nu\delta$ and that $-\nu\delta > \bar{\theta}^A_\lambda \geq \bar{\theta}^A_\lambda s_A + \bar{\theta}^B_\lambda s_B$ (see Appendix E for details), the share of abatement expenditure in the economy constantly grows until FA is achieved.

Optimal research regimes For a sufficiently low level of abatement expenditure in the economy, it is optimal to allocate all resources for research to improve the resource-saving technology. With the rising share of abatement expenditure, however, it becomes optimal to assign all researchers to improve the abatement technology when this share becomes sufficiently high. Thus, the economy switches from one research regime to another. The shift from resource-saving R&D to abatement-augmenting research happens before an FA regime succeeds a PA regime. This leads us to the following proposition:

Proposition 4 *There is a share of abatement expenditure $\hat{\sigma} \equiv \sigma(\hat{t}) < a\beta/b$ such that all scientists are allocated to improve the resource-saving technology if $\sigma < \hat{\sigma}$ and to improve the abatement-augmenting technology otherwise. A research regime in which there is an interior solution for scientific activity, that is, $s_B \in (0,1)$, can only hold when $\sigma = \hat{\sigma}$.*

Proof. The proof is provided in Appendix H. □

3.4.3 Transition dynamics

Transition dynamics of Z and λ Figure 2 demonstrates the phase plane for $(\theta_Z, \theta_\lambda)$ by plotting the isoclines $\dot{\theta}_Z = 0$ and $\dot{\theta}_\lambda = 0$. In the figure, $\dot{\theta}^B_Z = 0$ stands for the isocline $\dot{\theta}_Z = 0$ in an abatement-augmenting research regime:

$$\kappa(\sigma)\frac{\dot{\theta}_Z}{\theta_Z+\delta} = [\bar{\theta}^B_\lambda + \kappa(\sigma)\bar{\theta}^B_Z] - \theta_\lambda - \kappa(\sigma)\theta_Z,$$

Conversely, $\dot{\theta}^A_Z = 0$ stands for the isocline $\dot{\theta}_Z = 0$ in a resource-saving research regime (see Eq. 3.24). Since by hypothesis $\alpha b > a$ and $\kappa(0) = 1-\alpha+\alpha\epsilon$, the isocline $\dot{\theta}^A_Z = 0$ lies below the isocline $\dot{\theta}^B_Z = 0$ when $\sigma = 0$. When $\sigma = \alpha\beta$, however, the opposite is true. Notice also that the isocline $\dot{\theta}^B_Z = 0$ passes through the origin. For further discussion on the constraints and conditions limiting the transitional dynamics, we refer the reader to p. 10.

Since the share of expenditure on abatement in the economy constantly increases until FA is achieved, the isoclines $\dot{\theta}^A_Z = 0$ and $\dot{\theta}^B_Z = 0$ rotate counterclockwise and converge to the horizontal lines $\bar{\theta}^A_\lambda$ and $\bar{\theta}^B_\lambda$, respectively. As a result, the optimal trajectory moves in the southeast direction. Once, σ gets sufficiently high ($\hat{\sigma}$ in the figure), the sectoral social wages ω_A and ω_B are equalized, and the economy enters the abatement-augmenting research phase. The dynamics in this latter phase are dictated by the isoclines $\dot{\theta}_\lambda = 0$ given by Eq. (3.25) and the isocline $\dot{\theta}^B_Z = 0$ that converges to the horizontal line $\bar{\theta}^B_\lambda$.

In both regimes, the optimal trajectories move toward $(-\delta, -\nu\delta)$. Nonetheless, because θ_σ from Eq. (3.30) is greater than the one in a research-saving research regime, FA is attained earlier. Therefore, the phase with abatement-augmenting research corresponds to an accelerated approach toward $(-\delta, -\nu\delta)$.

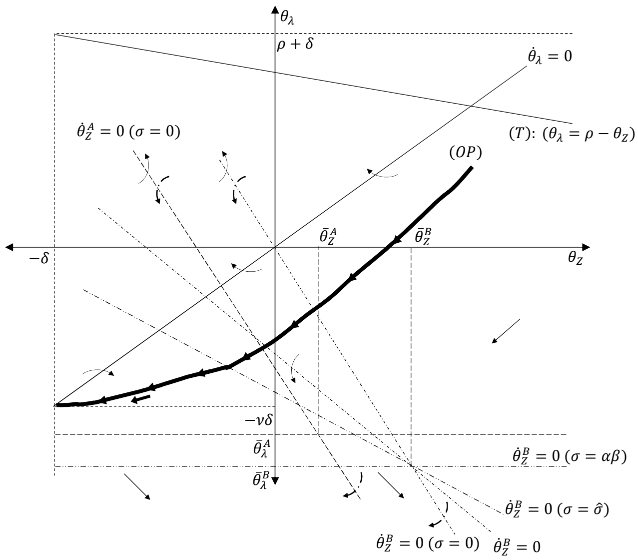

Fig. 2: *Phase diagram analysis*

Transition dynamics of the economic variables During the PA phase, the abatement-augmenting research regime provides interesting economic dynamics. Since the algebra is tedious, we relegate the calculations in this regard to Appendix J. The analysis shows that an early allocation of researchers to abatement-augmenting research sector can lead to a lower rate of economic growth than under a resource-saving research regime. Specifically, when the switch to the abatement-augmenting research regime happens for a sufficiently small share of abatement expenditure, the economic growth during the transition to the FA regime would be smaller than the one under a resource-saving research regime. Further calculations reveal that the growth rate of the abatement expenditure is always higher in an abatement-augmenting research regime. Accordingly, following the change in the research regime and during the transition to FA, the growth rate of consumption is smaller. Nonetheless, because the net opportunity cost of pollution decreases at a faster pace, that is, $\bar{\theta}^A_\lambda > \bar{\theta}^B_\lambda$, the transition to FA takes shorter than it would take in a resource-saving research regime.

Similar to our earlier discussions, the dynamics of y, z, and c can be non-monotonous when the growth rates of the social cost of pollution and the pollution stock, θ_λ and

θ_Z, take positive values during the PA phase. Since all researchers are allocated to improve the abatement technology when σ is sufficiently large, it is less likely to observe this type of non-monotonous behavior when the economy switches to an abatement-augmenting research regime.

Environmental Kuznets curve Depending on the initial conditions (discussed in Appendix K), there can be an initial phase in which the pollution stock increases; that is, $\theta_Z > 0$. The increase in the pollution stock can continue even when all resources for research are allocated to augment the abatement technology. Nonetheless, this requires the switch to the abatement-augmenting research regime to take place when σ is sufficiently low. This way, the economy can be on the optimal path that is located to the right of the θ_λ axis in Figure 2. When OP passes through the positive θ_Z axis, the pollution stock displays an EKC pattern.

Balanced growth path Once FA is attained, the economy settles on a balanced-growth path along which the economic growth is higher. The dynamics of the variables under a perpetual FA regime are straightforward. Since the share of the abatement expenditure in the total output attains its maximum value: $\sigma = \alpha\beta$, the pollution stock diminishes with the natural decay rate: $\theta_Z = -\delta$. Given $\theta_B \equiv \dot{B}/B = b$, we can calculate the growth rates for the variables from Eqs. (3.27) and (3.28). Accordingly, the growth rates of output, abatement expenditure, and consumption is $\theta_j = \alpha b/(1-\alpha\beta)$ for $j = y, x, c$. Further, the use of the polluting input and abatement grow by $\theta_i = b/(1-\alpha\beta)$ for $i = z, q$.

To close the model, we need to consider the initial conditions. We defer this analysis to Appendix K.

The following remark summarizes the main findings in this section:

Remark 4 When the innovative capacity in the abatement-augmenting R&D sector is sufficiently high,

- It is optimal to fully abate the pollution flow permanently while all resources for research are directed at improving the abatement technology if the initial net opportunity cost of pollution is smaller than the social cost of pollution. The output, resource use, consumption, and abatement all increase over time at constant rates.
- When the initial net opportunity cost of pollution is bigger than the social cost of pollution, the optimal path is composed of three phases:

a. During the first phase, the economy performs PA. The share of abatement in the economy rises over time. The resources for research are allocated to improve the resource-saving technology. The economic output, resource use, and consumption may experience non-monotonous evolutions. The pollution stock size either constantly declines or either first increases before having to decrease.
b. In the second phase, when the share of expenditure for abatement in the economy is sufficiently high, all resources for research are allocated to improve the abatement-augmenting technology. Therefore, the economy switches from one research regime to another. During this phase, the economic output, resource use, and consumption may sustain their non-monotonous evolutions. The pollution stock size either continually declines or increases before having to decline. The economic growth is slower than the corresponding rate in a resource-saving research regime until the share of abatement expenditure in the economy becomes sufficiently high. The growth rate of consumption is lower throughout this whole phase.
c. In the third phase, the economy fully abates its pollution while all resources for research are allocated to improve the abatement technology. With this phase, the economy settles on a BGP.

4 Discussion

Our findings mainly ensue from positing a trade-off between production and abatement, and in turn, between resource-saving and abatement-augmenting technical change. Accordingly, while the composite macro good production leads to pollution, an economy can partially or fully abate the pollution and direct its resources for either the resource-saving or abatement-augmenting research.

Our results indicate conditional path dependency. To be specific, when the innovative capacity in the abatement-augmenting research sector is sufficiently high, the optimal allocation of researchers depends on the initial level of the accumulated technical knowledge or pollution stock. The optimal allocation of researchers then boils down to the share of the abatement expenditure in the economy. For low levels of abatement expenditure, scientific activity takes place only in the resource-saving research sector. However, when the share of abatement expenditure in the economy is sufficiently high, it is optimal to allocate the resources for research to improve the abatement technology. Our results point to a slow down in economic and consumption growth during the transition to a pollution-free backstop production. Once the pollution is entirely abated, and fossil fuels are entirely replaced by backstop technologies, which are clean and rely on everlasting resources, the economy settles on a BGP.

On the other hand, when the innovative capacity of the resource-saving research sector is sufficiently high, the allocation of researchers is path-independent. Thus, all researchers are allocated to improve the resource-saving technology from the start.

The main reason for the sudden switch from the resource-saving research regime to the abatement-augmenting research regime owes to the linear formulation of the knowledge generation function. We could alternatively assume that the laws of motions for the two technologies were $\dot{A}(t) = as_A^{\varphi_A}(t)A(t)$ and $\dot{B}(t) = bs_B^{\varphi_B}(t)B(t)$ with φ_J $(J = A, B) \in (0, 1)$. To simplify the analysis, let $\varphi_J = \varphi$. An analysis similar to the one in Appendix A yields

$$s_A(t) = \frac{1}{1 + \left(\frac{\frac{b}{\beta} \int_t^\infty e^{-\rho(\tau - t)} \sigma(\tau) p(\tau) y(\tau) d\tau}{a \int_t^\infty e^{-\rho(\tau - t)} p(\tau) y(\tau) d\tau} \right)^{\frac{1}{1-\varphi}}}.$$

It is clear that this formulation avoids corner solutions. Because the abatement dynamics are similar to the ones when the growth equations are linear in the level of research, the share of abatement expenditure rises in time until FA is attained. With the rising share of abatement, the share of resources allocated to augment resource-saving research decreases. When $\sigma = \alpha\beta$ for $t \geq \bar{t}$, the steady-state level of research allocated to resource-saving technology can be calculated to yield:

$$s_A(t) = \frac{1}{1 + \left(\frac{\alpha b}{a} \right)^{\frac{1}{1-\varphi}}}.$$

Consequently, the higher the innovative capacity in the abatement-augmenting research, the lower is the steady-state share of resource-saving R&D. Notice that $\lim_{\varphi \to 1} s_A(t) = 1$ if $\alpha b < a$ and $\lim_{\varphi \to 1} s_A(t) = 0$ if $\alpha b > a$, thus, confirming the outcomes of the original analysis. (The result is indeterminate for $\alpha b = a$.)

Although it constitutes a more general framework, we refrained from this non-linear formulation in our analysis for two reasons. First of all, in the recent literature on the endogenous technical change, the growth equations are linear in the level of research [see, e.g., 4, 47, 33, 62]. We relied on this formulation to understand whether a two-sector model could lead to results that demonstrated differences with the ones in the endogenous technical change literature. Secondly, concavity in the level of research introduces two new parameters, which necessitate further discussion and justification. As of yet, we are not aware of another study in the corresponding literature that employed this formulation. We believe that such a deviation is important, but considering the particular focus in and length of our analysis, we believe that this deviation should be analyzed in a new study.

Our focus on a bi-sectoral model of technical change allows us to obtain results which are in contrast with the recent literature on endogenous technical change and the environment [see 4, 49, 5, 6, 10]. This literature mainly employs a one-sector model in which a final good is produced using polluting and clean intermediates, and accordingly, focuses on the trade-off between dirty and clean input production and technologies. Under this trade-off, an economy progressively directs its research toward the technology with higher accumulated technological knowledge. Thus, there is "path dependence" in the direction of technical change [6, 10].

As an example, consider Acemoglu et al [4]. In a one-sector economy, a final good is produced using clean and dirty (thus, polluting,) intermediates that are substitutes. Similar to our model, growth equations are linear in the level of research and a limited number of researchers can be allocated to improve the two technologies. The optimal policy dictates the allocation of researches to improve the clean intermediate technology (with the use of carbon tax and research subsidies in a decentralized setting) until this technology gets sufficiently advanced and the clean intermediate in the final good production attains a sufficiently large share.[15] Although it affects the duration and level of the carbon tax and research subsidies, the difference between the innovative capacities does not alter the outcome. Eventually, all researchers are allocated to augment the clean technology, and the economy grows proportional to the clean research sector's innovative capacity. This is the case even if the innovative capacity is significantly smaller than the one for the dirty technology. As we show with our two-sector model, an economy does not have to be constrained by such a low growth rate when abatement is done out of the production sector.

In addition to the differences that it can cause, our formulation of a two-sector model also enables us to demonstrate the critical role that resource-saving technical change can play in dealing with a pollution problem. Because such a multi-sectoral approach is missing in the literature on endogenous technical change and environment, the importance of resource-saving technical change in reducing environmental pollution has been underrated.

4.1 A note on no technical progress

Before concluding our study, we would like to provide some basic insights into the optimal dynamics and trade-offs between economic activity and the environment when there is no technological progress. This section allows us to demonstrate how technical progress can fundamentally affect the dynamics of the pollution stock and the economic dynamics. In contrast to the analysis in Section 3, where an FA policy would be pursued permanently, for an economy without any technical progress, the optimal policy would be to permanently let the pollution accumulate until it reaches its steady-state value, $\hat{Z}$.

[15] Analogous to Corollary 2, all researchers are allocated to improve the clean (intermediate) technology for a sufficiently high level of pollution stock or a sufficiently advanced clean technology.

4.2 Transition dynamics

Transition dynamics of Z and λ Supposing that $a = b = 0$, and therefore, that there is no technical change, Figure 3 depicts the phase plane for (Z, λ) plotting the isoclines $\dot{\lambda} = 0$ and $\dot{Z} = 0$. (We relegate the calculations to Appendix L.) Inspection of the figure indicates that the optimal trajectory is saddle-path stable and converges towards the long-run steady state $(\hat{Z}, \hat{\lambda})$, which is defined as the simultaneous solution of $\dot{\lambda} = 0$ and $\dot{Z} = 0$. If the initial pollution stock is smaller than $\hat{Z}$ and the social cost of pollution is lower than $\hat{\lambda}$, both the pollution stock and cost of pollution increase over time. In this initially low pollution stock scenario, the optimal policy is to permanently let the pollution accumulate until it reaches its steady-state level, $\hat{Z}$, over time. This induces a permanent rise in the opportunity cost of pollution, or the so-called "social cost of pollution," which approaches its long-run steady-state level, $\hat{\lambda}$. As λ is an increasing function of σ, the share of GDP devoted to abatement increases and converges toward a unique long-run value, $\hat{\sigma}$, which solves $\lambda(\sigma) = \hat{\lambda}$ and corresponds to PA. Although the share of the abatement expenditure in the GDP increases over time, this does not prevent the pollution stock from growing over time until it reaches its steady-state value, $\hat{Z}$.

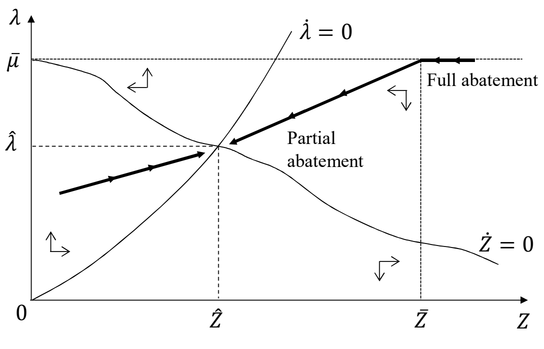

Fig. 3: *Phase diagram analysis*

Transition dynamics of the economic variables In light of the monotonic relationship between σ and λ, z and x can also be deemed as functions of λ (see Eqs. (3.27) and (3.28)). Accordingly, when the cost of pollution to the society increases, the use of the resource, and thus, the level of output, will get smaller. In contrast, the abatement expenditure, and thus, the abatement activity, will get higher. Conversely, when the cost of pollution to the society decreases, the resource use will increase, and the abatement expenditure will decrease.

Discussion Such an optimal behavior is very close to the findings of Moreaux and Withagen [50] for the abundant resource case.[16] The model is cast in a partial setting with either a constant unit or a convex cost of abatement. Our model is a general equilibrium model because of the linkage between production, consumption, and abatement decisions. The "cost" of abatement in our setting has to be understood in terms of sacrificing consumption. This is well reflected in Eq. (3.5), which depicts equality between the social marginal value of abatement input, x, and its cost in terms of "consumption sacrifice" evaluated in number of "utils." Our study shows that the main conclusion of Moreaux and Withagen [50] remains valid in this more general setup.

As is demonstrated in Section 3, things turn out very differently when we allow for technical change. For an economy with no technical progress, the optimal policy is to permanently let the pollution accumulate until it reaches its steady-state value, $\hat{Z}$ (see Figure 3). This induces a permanent rise in the social cost of pollution, which eventually reaches its long-run level, $\hat{\lambda}$. With continuous R&D efforts directed at improving resource use efficiency, however, the rise in the pollution stock is not permanent, and the economy exhibits an EKC-like behavior. To sum up, the initial decline in consumption and rise in the pollution stock will be followed by a permanently rising level of consumption and diminishing stock of pollution. Thus, the short-run losses in the social welfare are small compared to the long-run gains.

5 Conclusion

Economic growth will be beneficial only if it does not undermine the ecological basis of our civilization. Achieving this, however, presents major challenges, requiring significant changes in the composition of the global GDP, and more emphasis on technological progress. A relevant question then is whether we can expect green growth or expect the economy to settle on a BGP, where the pollution stock converges to zero? If so, how can this be achieved, and what can be the role of technological progress?

To shed light on these questions, we used a two-sector economic growth model with resource-saving and abatement-augmenting technical change. Our results indicate that green growth is always socially optimal. Despite this monolithic result, time paths of the optimal solution trajectories can be different depending on the model's fundamentals. Once the pollution is fully abated eventually, the fossil fuels are entirely replaced by backstop technologies, which are clean and rely on everlasting resources. Furthermore, we found that researchers are optimally allocated to the sector with sufficiently high innovative capacity. In particular, when

[16] Moreaux and Withagen [50] also highlight the aftermath of resource scarcity, which is the main contribution of the paper.

the innovative capacity in the abatement-augmenting research sector is sufficiently high, the optimal allocation of researchers depends on initial values. Otherwise, the allocation of researchers is path-independent.

In addition to the analytical and theoretical discussions, we supplemented our analysis with phase diagram analyses. In particular, we provided the conditions and illustrations of the time profiles of economic variables that exhibit non-monotonous behaviors during transitions from pollution-intensive to pollution-free backstop production. Moreover, while the economy grows greener, we showed that, depending on the initial conditions, the pollution stock either constantly declines or first increases before having to decrease. When the latter occurs, the pollution stock displays a pattern resembling an EKC.

Our findings mainly ensue from positing a trade-off between production and abatement, and thus, between resource-saving and abatement-augmenting technical change. To the best of our knowledge, we are not aware of any other studies that study the dynamics involved in sustainable green growth using a bi-sectoral model of production and abatement, where both sectors benefit from factor-augmenting technical change. In contrast, the recent literature on endogenous technical change and the environment solely focuses on the trade-off between dirty and clean input production and technologies. In this regard, sustainable growth necessitates resources to be allocated progressively from dirty to clean production and technology. Nevertheless, our results indicate conditional path-dependency. Lastly, by using a two-sector model, we address a lack of attention to multi-sector growth models in neoclassical growth theory [67]. While the neoclassical growth theory is about the evolution of potential output, endogenous technical change is about the evolution of research while an estimated level of growth rate is attained in the long run. By using a two-sector model, rather than a one-sector model in which a final good is produced using polluting and clean intermediates, we show that different results and transitional dynamics can emerge depending on which sector R&D is directed at and how much resources are allocated to carry out research in each sector.

In this paper, we confined our attention to a social welfare maximization problem. To implement the first-best outcome in a decentralized economy, two market imperfections will need to be addressed. First, there will be a tax on the polluting activity. Second, the research activities will need to be subsidized. This is because the social and private values of an innovation generally differ on account of the several distortions that researchers encounter.[17] Additionally, it is possible to calibrate our model with data, and then numerically calculate the first-best tax and subsidy policies, followed by an assessment of the welfare implications of alternative tax and subsidy policies.

Lastly, to provide analytical characterizations of the optimal dynamics, we abstracted from physical capital and made the simplifying linearity assumption in the knowledge generation function in our study. The latter assumption allowed us to discern whether a two-sector model could lead to outcomes that are in contrast to the ones in the endogenous technical change literature. Consequently, our two-sector model analysis can be further developed by embodying technical change in physical capital and assuming a non-linear R&D specification.

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[17] To name a few, these distortions are (i) the intertemporal spillover effects (i.e., inventors do consider the fact that the ideas they produce can be used to generate new innovations), (ii) the appropriability effects (i.e., inventors can only partially appropriate the social value they create), and (iii) the creative- destruction effect (Grimaud et al., 2011).

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Appendix A Proof of Theorem 1

Proof. The transversality condition (TC) states that $\lim_{t\to\infty} e^{-\rho t}\lambda(t) Z(t) = 0$ which is equivalent to $-\rho + \theta_\lambda + \theta_Z < 0$ being evaluated at the limit $t \to \infty$.

From Eq. (3.21) the TC can be restated as

$$\frac{n}{Z} - \frac{Z^\nu}{\lambda} < 0.$$

However, the solution to $\dot{Z} = n - \delta Z$ is given by

$$Z = \frac{\int e^{\delta t} n dt + Z_0}{e^{\delta t}}.$$

This implies

$$\frac{n}{Z} = \frac{e^{\delta t} n}{\int e^{\delta t} n dt + Z_0}.$$

Deduce that n/Z is the growth rate of

$$\int e^{\delta t} n dt + Z_0,$$

which is bounded from below by δ if n does not converge to zero.

Likewise, the solution to $\dot{\lambda} = (\rho + \delta)\,\lambda - Z^{\nu}$ is

$$\lambda = \frac{-\int e^{-(\rho+\delta)t} Z^{\nu} dt + \lambda_0}{e^{-(\rho+\delta)t}}$$

implying

$$\frac{Z^{\nu}}{\lambda} = \frac{e^{-(\rho+\delta)t} Z^{\nu}}{\lambda_0 - \int e^{-(\rho+\delta)t} Z^{\nu} dt}.$$

It follows that $-Z^{\nu}/\lambda$ is the growth rate of

$$\lambda_0 - \int e^{-(\rho+\delta)t} Z^{\nu}.$$

Nevertheless, $\lambda \geq 0$ implies

$$\lambda_0 \geq \int e^{-(\rho+\delta)t} Z^{\nu}. \tag{A.1}$$

Note that $e^{-(\rho+\delta)t} Z^{\nu}$ is zero or negative in the limit. Otherwise, $\int e^{-(\rho+\delta)t} Z^{\nu}$ would diverge to infinity violating Eq. (A.1). Thus Z^{ν}/λ converges to zero, which implies that

$$\frac{n}{Z} - \frac{Z^{\nu}}{\lambda} \tag{A.2}$$

converges to δ, thus contradicting the TC. Then $n \to 0$.

Recall that $\sigma \equiv \frac{x}{y} = \alpha\beta \frac{q}{z}$. As $n \equiv z - q = z(1 - q/z)$, if $n \to 0$, then $q/z \to 1$ as $z > 0$ for all t. On the basis of this result, we conclude that $\sigma \to \alpha\beta$. □

Appendix B Proof of Proposition 1

Proof. The optimal research policy at time t depends on the marginal social value of doing research in the two R&D sectors. Multiplying both sides of Eqs. (3.10) and (3.11) by $e^{-\rho t} A$ and $e^{-\rho t} B$, respectively, and rearranging yields

$$(e^{-\rho t} \dot{\lambda_A A}) = -e^{-\rho t} p A z^{\alpha} \text{ and}$$
$$(e^{-\rho t} \dot{\lambda_B B}) = -e^{-\rho t} \mu B x^{\beta}.$$

Time integrating these two equations and taking into account the transversality conditions for the technology indexes A and B given by Eq. (3.12) allows us to write the sectoral (social) wages as

$$\omega_A(t) \equiv a\lambda_A(t) A(t) = \int_t^{\infty} e^{-\rho(\tau - t)} a p(\tau) y(\tau) d\tau, \tag{B.1}$$

$$\omega_B(t) \equiv b\lambda_B(t) B(t) = \int_t^{\infty} e^{-\rho(\tau - t)} b \mu(\tau) q(\tau) d\tau, \tag{B.2}$$

where $\omega_A(t)$ and $\omega_B(t)$ are the social wages in the resource-saving and abatement-augmenting research sectors, respectively. If $\omega_A(t) > \omega_B(t)$, all resources for research will be allocated to improve the resource-saving technology at time t. To determine this, we write $\omega_A(t) - \omega_B(t)$ using Eq. (3.5) as

$$\omega_A(t) - \omega_B(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \frac{\beta \mu(\tau) q(\tau)}{\sigma(\tau)} \left(a - \frac{b\sigma(\tau)}{\beta} \right) d\tau. \tag{B.3}$$

Because the FA constraint necessitates that $\sigma \leq \alpha\beta$, if $a > \alpha b$, then $\omega_A(t) - \omega_B(t) > 0$ for all t. □

Appendix C Proof of Proposition 2

Proof. As, by hypothesis, there is FA over $[\bar{t}, \infty)$, the pollution stock will shrink at the constant rate of δ; that is, $\theta_Z = -\delta$. Integrating the equation of motion for the social cost of pollution (i.e., Eq. (3.9)) over $[\bar{t}, \infty)$ yields

$$\lambda(t) = e^{(\rho+\delta)(t-\bar{t})} \left[\lambda(\bar{t}) - Z(\bar{t})^{\nu} \frac{1 - e^{-(\rho + (1+\nu)\delta)(t - \bar{t})}}{\rho + \delta(1+\nu)} \right] \tag{C.1}$$

When FA is practiced from $\bar{t}$ onwards, Eq. (C.1) yields the time path for the cost of pollution. Taking into account the transversality condition $\lim_{t\to\infty} e^{-\rho t} \lambda(t) Z(t) = 0$, the social cost of pollution over $[\bar{t}, \infty)$ simplifies to

$$\lambda(t) = \frac{Z(\bar{t})^{\nu} e^{-\nu\delta(t - \bar{t})}}{\rho + \delta(1+\nu)}, \tag{C.2}$$

whose growth rate is $\theta_{\lambda} = -\nu\delta$.

As for $\theta_{\bar{\mu}}$, note that

$$M(A(t), B_0) = \alpha(\alpha\beta)^{-\frac{1-\alpha+\alpha\epsilon}{1-\alpha\beta}} \left(A(t)^{1-\beta-\epsilon} B_0^{-(1-\alpha+\alpha\epsilon)} \right)^{\frac{1}{1-\alpha\beta}},$$

whose growth rate is

$$a(1 - \beta - \epsilon)/(1 - \alpha\beta).$$

By definition, $\bar{\mu}(t) \equiv M(A(t), B_0)\, \bar{\varphi}_{\lambda}$. Thus, the growth rate of $\bar{\mu}$ is equal to that of $M(A(t), B_0)$. This completes the proof. □

Appendix D Proof of Proposition 3

Proof. If $\bar{\mu}(0) > \lambda(0)$, then $n(0) > 0$, that is, PA is optimal at time $t = 0$. However, $-\delta\nu > a(1-\beta-\epsilon)/(1-\alpha\beta)$ implies $\theta_{\bar{\mu}} < \theta_{\bar{\lambda}}$, where $\bar{\lambda}$ denotes the social cost of pollution when the pollution flow is fully abated. Accordingly, the growth rate of $\bar{\lambda}$ remains higher than that of $\bar{\mu}$ at all times. Thus, after a certain point in time, that is, when $t = \bar{t} > 0$, $\bar{\mu}(\bar{t}) = \lambda(\bar{t})$. Consequently, $n(t) = 0$ for $t \geq \bar{t}$. □

Appendix E Proof of Corollary 1

By hypothesis, $\alpha b > a$ and $\epsilon > 1$. These two inequalities imply that $\bar{\theta}_{\lambda}^{A} > \bar{\theta}_{\lambda}^{B}$, where $\bar{\theta}_{\lambda}^{B} \equiv -b(1-\alpha+\alpha\epsilon)/(1-\alpha\beta)$. During FA, the net opportunity cost of pollution shrinks at the rate of $\bar{\theta}_{\lambda}^{A}s_A + \bar{\theta}_{\lambda}^{B}s_B$. Since $\bar{\theta}_{\lambda}^{A} > \bar{\theta}_{\lambda}^{B}$, any convex combination of $\bar{\theta}_{\lambda}^{A}$ and $\bar{\theta}_{\lambda}^{B}$ is smaller than $\bar{\theta}_{\lambda}^{A}$. Hence, when $\bar{\mu}(\bar{t}) \leq \lambda(\bar{t})$ and considering that research can take place in both sectors, $\bar{\theta}_{\lambda}^{A}s_A + \bar{\theta}_{\lambda}^{B}s_B (\leq \bar{\theta}_{\lambda}^{A})$, the net opportunity cost of pollution always stays lower than the social cost of pollution if $-\delta\nu > \bar{\theta}_{\lambda}^{A}$. Consequently, $-\delta\nu > \bar{\theta}_{\lambda}^{A}$ provides a sufficient condition for FA to be pursued permanently from $\bar{t}$ onward.

Appendix F Proof of Corollary 2

Proof. Given that $-\delta\nu > \bar{\theta}_{\lambda}^{A} > \bar{\theta}_{\lambda}^{B}$ and, therefore, $\theta_{\bar{\mu}} < \theta_{\bar{\lambda}}$, the marginal cost of fully abating the pollution will always be lower than its social cost from time $\bar{t}$ onward; that is, $\bar{\mu}(t) \leq \lambda(t)$ and, in turn, $\sigma(t) = \alpha\beta$ for $t \geq \bar{t}$. The difference between the social wages in the two research sectors during the FA regime can be written as

$$\omega_B(\bar{t}) - \omega_A(\bar{t}) = \int_{\bar{t}}^{\infty} e^{-\rho(\tau-\bar{t})} \frac{\beta\mu(\tau)q(\tau)}{\alpha\beta} (\alpha b - a)\, d\tau,$$

where we substitute $\alpha\beta$ for σ in Eq. (B.3). Since $\alpha b > a$, the social wage in the abatement-augmenting research sector is higher for $t \in [\bar{t}, \infty)$.

Recall that $\bar{\mu}(t) \equiv M\big(A(t), B(t)\big)\bar{\varphi}_{\lambda}$ and $\lambda(t) = Z(t)^{\nu}e^{-\nu\delta(t-\bar{t})}/\big(\rho+\delta(1+\nu)\big)$. If $M\big(A(0), B(0)\big)\bar{\varphi}_{\lambda} \leq Z(0)^{\nu}/\big(\rho+\delta(1+\nu)\big)$ and $-\delta\nu > \bar{\theta}_{\lambda}^{A}$, the pollution flow is fully abated, and all scientists are directed at the B sector from $t = 0$ onward.

□

Appendix G Proof of Corollary 3

Proof. When the net opportunity cost of pollution is bigger than the social cost of pollution, that is, $\bar{\mu}(0) > \lambda(0)$, then PA is optimal at time $t = 0$. Thus, $n(0) > 0$. However, $-\delta\nu > \bar{\theta}_{\lambda}^{A} \geq \bar{\theta}_{\lambda}^{A}s_A + \bar{\theta}_{\lambda}^{B}s_B$ (see Appendix E for details) implies $\theta_{\bar{\mu}} < \theta_{\bar{\lambda}}$. Accordingly, the growth rate of $\bar{\lambda}$ remains higher than that of $\bar{\mu}$ at all times. Thus, after a certain point in time, that is, when $t = \bar{t} > 0$, the net opportunity cost of pollution gets equal to the social cost of pollution $\bar{\mu}(\bar{t}) = \lambda(\bar{t})$. Consequently, FA is pursued permanently for $t \geq \bar{t}$. □

Appendix H Proof of Proposition 4

Proof. From Eq. (3.30), $\theta_{\sigma} > 0$ until FA is attained. Since $\theta_{\sigma} > 0$, it is optimal to allocate all resources to the abatement-augmenting research for $t > \tilde{t}$, where $\sigma(\tilde{t}) = a\beta/b$. Thus, $\omega_A(t) - \omega_B(t)$ for $t > \tilde{t}$, where $\omega_A(t) - \omega_B(t)|_{t>\tilde{t}}$ equals the finite sum

$$\int_{\tilde{t}}^{\bar{t}} e^{-\rho(\tau-t)} \frac{\beta\mu(\tau)q(\tau)}{\sigma(\tau)} \left(a - \frac{b\sigma(\tau)}{\beta}\right) d\tau - \frac{(\alpha b - a)A(\bar{t})^{\frac{2-\beta-\epsilon}{\alpha\beta}} B(\bar{t})^{-\frac{\alpha(\epsilon-1)}{1-\alpha\beta}}}{\Big((\alpha\beta)^{\frac{\alpha\beta}{1-\alpha\beta}}(1-\alpha\beta)\Big)^{\epsilon-1}\big(\rho(1-\alpha\beta)+\alpha b(\epsilon-1)\big)} < 0.$$

Depending on the initial conditions $(A(0), Z(0), B(0))$ (see Appendix K), the intersection of $d\theta_{\lambda}/d\theta_Z$ with the optimal trajectory defines the optimal $(\theta_{\lambda}(0), \theta_Z(0))$. Then, given the initial values for the state variables, $\lambda(0)$ and $\sigma(0)$ can be determined from $\lambda(0) = Z(0)^{\nu}/\left(\rho+\delta-\theta_{\lambda}(0)\right)$ and

$$\frac{\sigma(0)^{-(1-\beta)\bar{\theta}_{\lambda}^{B}/b}}{(1-\sigma(0))^{\epsilon}} = \frac{\lambda(0)}{M(A(0), B(0))},$$

respectively.

Depending on the initial conditions, if $\sigma(0)$ is sufficiently low such that $a - b\sigma(0)/\beta > 0$ and $\omega_A(0) - \omega_B(0)|_{t>0} > 0$, then $s_A(0) = 1$. Because $\omega_A(t) - \omega_B(t)|_{t>\tilde{t}} < 0$, an abatement-augmenting research regime has to begin before $\hat{t}(< \tilde{t})$.

A mixed-research regime, in which resources are allocated to both research sectors, cannot be maintained because $\theta_{\sigma} > 0$ (cf. Eq. (3.30)) until the time when $\sigma(\bar{t}) = \alpha\beta$. The necessary condition for a mixed-research regime at time $\hat{t}$ is $\omega_A(t) - \omega_B(t) = 0|_{t=\hat{t}}$. Sustaining this regime necessitates

$$\frac{d\big(\omega_A(t) - \omega_B(t)\big)}{dt}|_{t=\hat{t}} = 0,$$

and consequently, $\sigma = a\beta/b$. Nevertheless, because $\theta_{\sigma} > 0$, the economy switches from one research regime to the other one.

In Appendix I, we conduct further analysis and show that an interior solution for scientific activity that can permanently be sustained cannot be part of an optimal solution. □

Appendix I Mixed-research regime

From Eqs. (3.7) and (3.8), an interior solution for the scientific activity requires

$$\omega_A \equiv a\lambda_A A = \omega_B \equiv b\lambda_B B = \omega,$$

where $\omega_A(t)$ and $\omega_A(t)$ are the social wages in the resource-saving and abatement-augmenting research sectors, respectively. Multiplying both sides of Eqs. (3.10) and (3.11) by $e^{-\rho t}A$ and $e^{-\rho t}B$, respectively, and rearranging yields

$$(e^{-\rho t}\dot{a\lambda_A A}) = e^{-\rho t}aApz^{\alpha} \text{ and}$$
$$(e^{-\rho t}\dot{b\lambda_B B}) = e^{-\rho t}bB\mu x^{\beta}.$$

The equality of social wages allows the equalization of these two expressions and gives us $apy = b\mu q$. Using this relationship and Eq. (3.5) yields $\sigma^M = \beta\frac{a}{b}$, which is, indeed, a constant. Thus, when there is research in both sectors, and the two sectors are active over $[0, \infty)$, the share of the abatement expenditure in the GDP is fixed. The fact that abatement cannot exceed the pollution flow, that is, $\sigma \leq \alpha\beta$, leads to $\sigma^M \leq \alpha\beta \implies a \leq \alpha b$. Therefore, to justify a mixed research regime, the innovative capacity b has to be sufficiently higher than a. Further, as, by hypothesis, $b > a/\alpha$, then, $\sigma^M < \alpha\beta$; that is, a permanent mixed-research regime in which the social sectoral wages are equal can only occur when the pollution is always partially abated.

As σ is parametrically determined, μ and n become functions of the technology indexes A and B only (see Eqs. (3.16) and (3.17)). From Eqs. (3.16) and (3.17), we can calculate the growth rate for the social cost of pollution and the net level of pollution as $\theta_\lambda = \frac{\bar{\theta}^A_\lambda}{a}\theta_A + \frac{\bar{\theta}^B_\lambda}{b}\theta_B$ (or, equivalently, $\theta_\lambda = \bar{\theta}^A_\lambda s_A + \bar{\theta}^B_\lambda s_B < 0$) and $\theta_n = \frac{\bar{\theta}^A_Z}{a}\theta_A + \frac{\bar{\theta}^B_Z}{b}\theta_B$ (or, $\theta_n = \bar{\theta}^A_Z s_A + \bar{\theta}^B_Z s_B > 0$), respectively. Accordingly, time differentiating the dynamic system given by Eq. (3.21) yields

$$\frac{\dot{\theta}_\lambda}{\rho + \delta - \theta_\lambda} = \frac{\bar{\theta}^A_\lambda}{a}\theta_A + \frac{\bar{\theta}^B_\lambda}{b}\theta_B - \nu\theta_Z, \tag{I.1}$$
$$\frac{\dot{\theta}_Z}{\theta_Z + \delta} = \frac{\bar{\theta}^A_Z}{a}\theta_A + \frac{\bar{\theta}^B_Z}{b}\theta_B - \theta_Z. \tag{I.2}$$

Following this, the isoclines $\dot{\theta}_\lambda = 0$ and $\dot{\theta}_Z = 0$ are

$$\theta_\lambda = \nu\theta_Z \text{ and} \tag{I.3}$$
$$\theta_Z = \frac{\bar{\theta}^B_Z - \bar{\theta}^A_Z}{\bar{\theta}^A_\lambda - \bar{\theta}^B_\lambda}\bar{\theta}^B_\lambda + \bar{\theta}^B_Z - \frac{\bar{\theta}^B_Z - \bar{\theta}^A_Z}{\bar{\theta}^A_\lambda - \bar{\theta}^B_\lambda}\theta_\lambda. \tag{I.4}$$

The dynamics of θ_λ and θ_Z in a mixed research regime are pictured in Figure 4. It can be seen that the isocline $\dot{\theta}_Z = 0$ is the segment joining the points $(\bar{\theta}^A_Z, \bar{\theta}^A_\lambda)$ and $(\bar{\theta}^B_Z, \bar{\theta}^B_\lambda)$. It is immediately verified that if $b > a/\alpha$, $\bar{\theta}^A_\lambda > \bar{\theta}^B_\lambda$. Furthermore, $\sigma^M < 1$ implies that $\beta a < b$. Thus, $\bar{\theta}^A_Z < \bar{\theta}^B_Z$. As $\theta_A = as_A$ and $\theta_B = bs_B$, $\theta_\lambda = \bar{\theta}^A_\lambda s_A + \bar{\theta}^B_\lambda s_B$. Then, the positivity constraint $s_A \geq 0$ is equivalent to $\theta_\lambda \geq \bar{\theta}^B_\lambda$. Moreover, $1 \geq s_A$ implies $\theta_\lambda \leq \bar{\theta}^A_\lambda$. Therefore, in a mixed-research regime $\theta_\lambda \in [\bar{\theta}^B_\lambda, \bar{\theta}^A_\lambda]$.

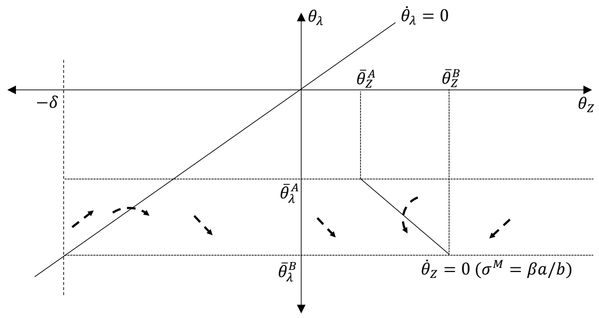

Fig. 4: *Mixed research regime: phase diagram analysis*

We see that all trajectories converge in finite time either toward the abatement efficiency, B, border (i.e., the horizontal line for $\bar{\theta}^B_\lambda$), or toward the production efficiency A border (i.e., the $\bar{\theta}^A_\lambda$ horizontal). It is, therefore, impossible to permanently sustain a mixed research regime. This is also justified by Theorem 1 stating that the share of the abatement expenditure will eventually converge to its upper bound $\alpha\beta$, thus, violating $\sigma = \beta\frac{a}{b}$ in a mixed-research regime.

In a mixed regime, the dynamics of the resources allocated for research is determined by the following equation:

$$s_A = \frac{\theta_\lambda - \bar{\theta}^B_\lambda}{\bar{\theta}^A_\lambda - \bar{\theta}^B_\lambda}. \tag{I.5}$$

Because $ds_A/d\theta_\lambda > 0$, the resources allocated for the resource-saving research increases when θ_λ approaches the $\bar{\theta}^A_\lambda$. Conversely, the resources allocated for abatement-augmenting research increases when θ_λ approaches the $\bar{\theta}^B_\lambda$.

Because $-\delta\nu > \bar{\theta}^B_\lambda$, σ steadily grows until FA is attained (i.e., when $\sigma(t) = 0$). During the PA phase a mixed-research regime which can be sustained cannot exist. During the FA phase, a mixed research regime cannot exist because the social wage in the abatement-augmenting research sector would always be higher. Instead, it is only when the pollution stock, or low level of resource-saving or abatement-augmenting technologies, are sufficiently low that the social planner would find it optimal partially abate the pollution and start allocating resources to augment the resource-saving technology (cf. Eq. (B.3)). Nevertheless, because $-\delta\nu > \bar{\theta}_\lambda^A$, the share of the expenditure in the economy that is spent on abatement increases in time. This can be observed from Eq. (3.23). When σ gets sufficiently high, and before abatement is fully abated, the resources for research are allocated to improve only the abatement-augmenting technologies. This phase starts right after the instant when $\omega_A(t) = \omega_B(t)$.

Appendix J Economic dynamics during a PA phase

Using Eqs. (3.27), (3.28) and (3.30), the difference between the output growth rates in the two regimes can be calculated to yield

$$\theta_y^A - \theta_y^B = \frac{a - \alpha b}{1 - \alpha\beta} - \frac{\alpha(1-\beta)}{(1-\alpha\beta)K_\lambda(\sigma)}\left(\bar{\theta}_\lambda^B - \bar{\theta}_\lambda^A\right)$$

where θ_j^i denotes the growth rate of variable j in research regime $i \in \{A, B\}$. One can calculate that

$$\begin{aligned} &\theta_y^A - \theta_y^B > 0 \quad \text{if} \quad \sigma < \frac{(1-\beta)a}{a(1-\beta) + (\alpha b - a)}, \\ &\theta_y^A - \theta_y^B \leq 0 \quad \text{otherwise.} \end{aligned} \tag{J.1}$$

When the switch to the abatement-augmenting research regime happens for a sufficiently small share of abatement expenditure, the economic growth during the transition to the FA regime would be smaller than the one under a resource-saving research regime.

Furthermore, the growth rate of the abatement expenditure is always higher in an abatement-augmenting research regime:

$$\theta_x^A - \theta_x^B = \frac{a - \alpha b}{1 - \alpha\beta} + \frac{1-\alpha}{(1-\alpha\beta)K_\lambda(\sigma)}\left(\bar{\theta}_\lambda^B - \bar{\theta}_\lambda^A\right) < 0. \tag{J.2}$$

Accordingly, the growth rate of consumption is smaller:

$$\theta_c^A - \theta_c^B = \frac{-(1-\alpha\beta)\left[(1-\beta)a + (1-\alpha)b\sigma\right]}{(1-\beta)\bar{\theta}_\lambda^B/b + (1-\alpha)\bar{\theta}_\lambda^A/a}. \tag{J.3}$$

Appendix K Initial conditions

First, the dynamical system given by Eq. (3.21) can equivalently be written as

$$\begin{aligned} \rho + \delta - \theta_\lambda =& \frac{Z^\nu}{M(A, B_0)\varphi_\lambda(\sigma)}, \\ \theta_Z + \delta =& \frac{N(A, B_0)\varphi_Z(\sigma)}{Z}. \end{aligned} \tag{K.1}$$

Using Eqs. (3.16), (3.17), and (3.21), the derivatives of θ_λ and θ_Z with respect to σ yield

$$\begin{aligned} \frac{d\theta_\lambda}{d\sigma} =& \frac{(\rho + \delta - \theta_\lambda)K_\lambda(\sigma)}{\sigma}, \\ \frac{d\theta_Z}{d\sigma} =& -\frac{(\theta_Z + \delta)K_Z(\sigma)}{\sigma}. \end{aligned}$$

Eliminating σ, Eq. (K.1) implicitly defines $\theta_\lambda(\theta_Z; A, B, Z)$ such that

$$\frac{d\theta_\lambda}{d\theta_Z} = -\frac{\rho + \delta - \theta_\lambda}{\theta_Z + \delta}\kappa(\sigma) < 0. \tag{K.2}$$

For a given initial vector $(A(0), Z(0), B_0)$, the intersection of this curve with the optimal trajectory OP defines the optimal initial vector $(\theta_\lambda(0), \theta_Z(0))$. Given the initial pollution stock, $Z(0)$, the initial social cost of pollution can be determined from $\lambda(0) = Z(0)^\nu / (\rho + \delta - \theta_\lambda(0))$.

Appendix L No technical progress

Suppose that $a = b = 0$. Thus, $\dot{A} = \dot{B} = 0$, and $A(t) = A_0$ and $B(t) = B_0$ for all t. Owing to the monotonic relationship between σ and λ (see Eq. 3.20), σ can be deemed a function of λ. Substituting σ, in $n(\sigma)$ enables us to define$\eta(\lambda) \equiv n(\sigma(\lambda))$, which is a strictly decreasing function of λ. Because the support of λ is $(0, M\bar{\varphi}_\lambda]$, owing to the boundary values of σ which are 0 and $\alpha\beta$. $\eta(\lambda)$ varies between $+\infty$ and 0 as λ varies between 0 and $M\bar{\varphi}_\lambda$.

When there is no technological change, the optimal paths of Z and λ are the solution of the following autonomous dynamic system described by Eqs. (2.3) and (3.9):

$$\dot{\lambda} = (\rho + \delta)\,\lambda - Z^\nu \quad \text{and} \tag{L.1}$$

$$\dot{Z} = \eta(\lambda) - \delta Z. \tag{L.2}$$

Figure 3 depicts the phase plane for (Z, λ) by plotting the isoclines $\dot{\lambda} = 0$ and $\dot{Z} = 0$. First, the isocline $\dot{\lambda} = 0$ is defined by $\lambda = Z^\nu/(\rho + \delta)$, which gives an increasing relation between Z and λ. Furthermore, $\lambda \leq M\bar{\varphi}_\lambda$ since the co-state variable is non-negative; that is, $\gamma \geq 0$. Secondly, the isocline $\dot{Z} = 0$ is defined by $\eta(\lambda) = \delta Z$, which gives a strictly decreasing relationship between the social cost of pollution, λ, and pollution stock, Z. Thus, λ varies between $M\bar{\varphi}_\lambda$ and 0 when $Z \to 0$ and $Z \to \infty$, respectively