Energy Storage and Renewable Energy: An Economic Approach

Appendices (cont.)

C Numerical implementation of the model

C.1 Method description

I solve the dynamic stochastic decision problem by collocation method. In doing this, I approximate the value function by an approximant $\widetilde{V}(S)$ that is parameterized by and solved for a vector of parameters, $\boldsymbol{\beta}$.

Abstracting from intermittency, z, a function can be approximated by a combination of n linearly independent basis functions, $\{\psi_i\}_{i=0}^{n}$, and basis coefficients, $\{\beta\}_{i=0}^{n}$, where n represents the number of collocation points:

$$F(x) \approx \widetilde{F}(x) = \sum_{i=1}^{n} \beta_i \psi_i(x).$$

The interpolation problem in one dimension is then to find $\{\beta\}_{i=0}^{n}$ such that it satisfies the function F at n interpolation points.

In vector notation this can be written as the following:

$$F(\boldsymbol{x}) = \boldsymbol{\psi}(x)\boldsymbol{\beta},$$

where $\boldsymbol{\Psi}(\boldsymbol{x}) = [\psi_1(x)\ \psi_2(x)\ \psi_3(x)\ \ldots\ \psi_{n+1}(x)]$ is the Chebyshev Vandermonde matrix, $\boldsymbol{\beta} = [\beta_1\ \beta_2\ \beta_3\ \ldots\ \beta_{n+1}]'$ and $\boldsymbol{x} = [x_1\ x_2\ ...\ x_{n+1}]'$,

$$\boldsymbol{\Psi}(\boldsymbol{x}) = \begin{bmatrix} \psi_1(x_1) & \psi_2(x_1) & \psi_3(x_1) & \ldots & \psi_{n+1}(x_1) \\ \psi_1(x_2) & \psi_2(x_2) & \ldots & \ldots & \psi_{n+1}(x_2) \\ \psi_1(x_3) & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \psi_1(x_{n+1}) & \ldots & \ldots & \ldots & \psi_{n+1}(x_{n+1}) \end{bmatrix}.$$

Similarly, in approximating a value function, I search for a coefficient vector, $\boldsymbol{\beta}$, that ensures that the approximant satisfies the Bellman equation and the equilibrium conditions at the n collocation nodes (one can think of collocation nodes as discrete “states of the economy").

In the current energy consumption and storage problem, I approximate a bivariate function, $V(S, z)$, as the planner considers the amount of stored energy and weather conditions before taking decisions. Therefore, I need apply the collocation method solution strategy in a multidimensional setting (i.e., multidimensional interpolation).

I solve (12) which is simplified to give:[15]

$$V(S,z) = \max_{\{Q_d, S'\}} \left\{ \frac{\left(Q_d + z\bar{Q}_c + \phi S - S'\right)^{1-\gamma}}{1-\gamma} - c_d\left(Q_d\right) + \delta \mathbb{E}_z\left[V(S', z')\right] \right\}$$
$$\text{s.t } \; 0 \le S \le \bar{S},$$
$$\underline{Q}_d \le Q_d \le \bar{Q}_d.$$

I approximate the value function using Chebyshev polynomials.[16] In doing this I discretize z into K (z_k for $k = (1, 2, ...K)$) and S into n collocation nodes (S_i for $i = (1, 2, ..., n)$). I determine the basis function coefficients for each z and S. For n basis functions, there are going to be n basis coefficients, and given K different weather states, the computational problem is to solve for $K \cdot n$ coefficients. Let us denote these coefficients by $\boldsymbol{\beta} = [\boldsymbol{\beta}_1 \; \boldsymbol{\beta}_2 \; \ldots \; \boldsymbol{\beta}_K]$, where $\boldsymbol{\beta}_z = [\beta_{1,z} \; \beta_{2,z} \; \ldots \; \beta_{n,z}]'$.

For each state of the weather, z_k, and for each level of stored energy, S_i, the approximant is formed as follows:

$$V(S_i, z) \approx \widetilde{V}(S_i, z) = \sum_{j=1}^{n} \beta_{j,z} \psi_j(S_i)$$

Given $V(S_i, z)$, I need to form the approximant to $V(S_i', z_k')$ as well. In doing this for S_i and z_k, I need to compute the level for the stored energy in the period ahead, S', and energy generation today Q_d given the intervals $\bar{S} \le S \le \bar{S}$ and $\underline{Q}_d \le Q_d \le \bar{Q}_d$. Using these boundaries I construct a grid for fossil fuel energy and energy storage, $\{Q_{di,z_k,l}\}_{l=1}^{n}$ and $\left\{S'_{i,z_k,l}\right\}_{l=1}^{n}$:

$$\boldsymbol{Q}_{di,z_k} = \{Q_{di,z_k,1} \; Q_{di,z_k,2} \; \ldots \; Q_{di,z_k,n}\}$$
$$\boldsymbol{S}'_{i,z_k} = \{S'_{i,z_k,1} \; S'_{i,z_k,2} \; \ldots \; S'_{i,z_k,n}\}$$

[15] The optimization here is done wrt Q_d and S' instead of Q, S'. This is due to the need for assigning boundary values for fossil fuel energy generation and energy storage in the numerical calculations.

[16] Chebychev basis polynomials in combination with Chebychev interpolation nodes can yield extremely well-conditioned interpolation collocation equations that one can accurately and efficiently solve. For a discussion regarding Chebychev basis functions and nodes, I refer the reader to Judd (1992), Judd (1998) and Miranda and Fackler (2002).

Given the approximants of the value function, I have $(K \cdot n)$ equations in $(K \cdot n)$ unknowns:

$$\sum_{j=1}^{n} \beta_{j,z}\psi_j(S_i) = \max_{\{Q_d,S'\}} \left\{ \frac{\left(Q_{di,z_k,l} + z\bar{Q}_c + \phi S - S'\right)^{1-\gamma}}{1-\gamma} - c_d\left(Q_d\right) + \delta \sum_{k=1}^{K}\sum_{l=1}^{n} \omega_k \beta_{j,z_k}\psi_j(S'_{i,z_k,m}) \right\}_{l=1}^{n}$$

where in approximating the integral operation I replaced the continuous random variable z_k with its discrete counter part ω_k, the weight functions, ω_k, using Gaussian quadrature scheme.[17] The weight functions are defined over the interval K. Quadrature nodes, here z_k, for $k = \{1, 2, ...K\}$, and corresponding quadrature weights ω_k, for $k = \{1, 2, ...K\}$ are selected such that $2K$ moments are satisfied.[18]

Above, I showed the approximant for $V(S'_i, z'_k)$ in its explicit form:

$$\begin{aligned} V(S'_i, z'_k) \approx \widetilde{V}(S'_i, z'_k) &= \sum_{j=1}^{n} \beta_{j,z_k}\psi_j(S'_{i,z_k,l}) \\ &= \sum_{j=1}^{n} \beta_{j,z_k} T_{j-1}\left[2\left(\frac{S'_{i,z_k,l} - \underline{S}}{\bar{S} - \underline{S}}\right) - 1\right] \text{ for } l = \{1, 2, \ldots, n\} \end{aligned}$$

where $\boldsymbol{\psi}_j(S'_{i,z_k}) = T_{j-1}\left[2\left(\frac{S'_{i,z_k,l} - \underline{S}}{\bar{S} - \underline{S}}\right) - 1\right]$ are the Chebyshev polynomial basis functions.

Having explained how the polynomial interpolation can work, I now explain the procedure of how to calculate the basis function coefficients, $\boldsymbol{\beta} = [\boldsymbol{\beta}_1\ \boldsymbol{\beta}_2\ \ldots\ \boldsymbol{\beta}_K]$. First I need to make a guess for the initial values of the basis functions' coefficients: $\boldsymbol{\beta}^0 = [\boldsymbol{\beta}_1^0\ \boldsymbol{\beta}_2^0\ \ldots\ \boldsymbol{\beta}_K^0]$. I then need to construct a grid of Chebyshev nodes, $\boldsymbol{u}_{n\times 1}$, and convert them into grid of stored energy, $\boldsymbol{S}$. The mapping looks like the following:

$$\boldsymbol{u} \rightarrow \boldsymbol{S} \in [\underline{S}, \bar{S}],\ \boldsymbol{S} = \frac{\bar{S} + \underline{S}}{2}\boldsymbol{I} + \frac{\bar{S} - \underline{S}}{2}\boldsymbol{u}$$

where $\boldsymbol{I}$ is a vector of ones, $\boldsymbol{I}_{n\times 1}$.

For $k = \{1, 2, \ldots, K\}$ and i={1,2,. . . ,n}, I construct a feasible grid of energy generation Q_d

[17] For a weight function defined on an interval K, $\int_K z\omega(z)dz \simeq \sum_{k=1}^{K} \omega_k z_k$.

[18] For the intermittent renewable energy production, $Q_c = z\bar{Q}_c$, its expected value can be calculated numerically as follows:

$$\mathbb{E}\left[Q_c\right] = \int_K z\bar{Q}_c\omega(z)dz \approx \sum_{k=1}^{K} \omega_m z_m \bar{Q}_c$$

and S_i' using Chebyshev nodes:

$$\boldsymbol{u} \to \boldsymbol{Q}_d \in [\underline{Q}_d, \bar{Q}_d],\ \boldsymbol{Q}_{di,z_k} = \frac{\bar{Q}_d + \underline{Q}_d}{2}\boldsymbol{I} + \frac{\bar{Q}_d - \underline{Q}_d}{2}\boldsymbol{u}$$
$$\boldsymbol{u} \to \boldsymbol{S}' \in [\underline{S}', \bar{S}'],\ \boldsymbol{S}'_{i,z_k} = \frac{\bar{S}}{2}(\boldsymbol{I} + \boldsymbol{u})$$

where the last equality resulted from $\underline{S} = 0$.

For $\boldsymbol{S}'$, I have the Chebyshev Vandermonde matrix: $\boldsymbol{\Psi}(\boldsymbol{S})$. Then

$$\widetilde{V}(\boldsymbol{S}, \boldsymbol{z}) = \frac{\left(\boldsymbol{Q}_{d,z} + \boldsymbol{z}\bar{Q}_c + \phi\boldsymbol{S} - \boldsymbol{S}'\right)^{1-\gamma}}{1-\gamma} - c_d\left(\boldsymbol{Q}_{d,z}\right) + \delta\sum_{k=1}^{K}\omega_k\boldsymbol{\Psi}(\boldsymbol{S}')\boldsymbol{\beta}_k^0$$

Taking the maximal entries in $\widetilde{V}(\boldsymbol{S}, \boldsymbol{z})$ I can construct $\tilde{\boldsymbol{V}}(\boldsymbol{\beta}^0)$ and update the coefficients according to Newton-Raphson method (see Judd (1998)):[19]

$$\boldsymbol{\beta}' = \boldsymbol{\beta} - \left[\boldsymbol{\Psi} - \widetilde{\boldsymbol{V}}^j(\boldsymbol{\beta})\right]^{-1}\left[\boldsymbol{\Psi}\boldsymbol{\beta} - \widetilde{\boldsymbol{V}}(\boldsymbol{\beta})\right]$$

where $\widetilde{\boldsymbol{V}}^j$ is the Jacobian of the approximant. One can then use the iterative update rule until the following difference gets to or smaller than a predetermined tolerance level, ϵ:

$$\boldsymbol{\beta}' - \left(\boldsymbol{\beta} - \left[\boldsymbol{\Psi} - \widetilde{\boldsymbol{V}}^j(\boldsymbol{\beta})\right]^{-1}\left[\boldsymbol{\Psi}\boldsymbol{\beta} - \widetilde{\boldsymbol{V}}(\boldsymbol{\beta})\right]\right) < \epsilon.$$

Long-run analysis After solving for the collocation coefficients, $\boldsymbol{\beta}$, I can estimate the evolution of the variables in the model. Using the grid I constructed for the stored energy $\boldsymbol{S}$, the solution to the model gives us an implicit policy rule: $\boldsymbol{S}' = g(\boldsymbol{S}, \boldsymbol{z})$.[20]

By satisfying the convergence criteria, I also solve for $\boldsymbol{S}'$. I can use these values to estimate the policy (transition) rule, hence solve for the Chebyshev function coefficients, $\boldsymbol{\phi}$:

$$\boldsymbol{S}' = \boldsymbol{\Psi}\boldsymbol{\phi} \to \boldsymbol{\phi} = \left(\boldsymbol{\Psi}'\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}'\boldsymbol{S}'$$

Using these coefficients one can pick a random sequence for weather conditions z_t for $t = 1, 2, \ldots, T$. One can then generate another sequence for S':

$$S_{t+1} = \boldsymbol{\Psi}(S_t)\boldsymbol{\phi}$$

Suppose that I do this N times (for N large) by generating N pseudorandom sequences for

[19]Where $\boldsymbol{\beta}^0$ was a guess for the initial values of the basis functions' coefficients

[20]Given S_i and z_k I now know what $S'_{i,k}$ is.

z.[21] Given the policy functions I calculated, $S'(S, z)$ and $Q_d(S, z)$, and the initial states S_0 and z_0, I can then generate a representative path from the N paths. Calculating the average value from the various pseudorandom sequences, one would get representative paths for the model variables in the long run. I will call this procedure a Monte Carlo Simulation.

C.2 Numerical implementation

I solve dynamic programming equation (2) by using collocation method and update the collocation coefficients according to the Newton's method (see Appendix C.1).[22] I construct a 40 Chebychev polynomial basis functions by forming 40 collocation nodes (4 nodes along S and 10 nodes along z dimension) and 40 basis function coefficients. The Beta distribution for the intermittent wind is approximated by Gauss-Legendre quadrature with 20 nodes.

The code is written in Matlab. I use CompEcon toolbox described in (Miranda and Fackler, 2002) in generating and evaluating the Chebychev polynomials, and doing the Monte Carlo simulations.

[21] Pseudorandom sequences are sequences that display some properties satisfied by random variables, such as zero serial correlation and correct frequency of runs, although none satisfy all properties of an i.i.d random sequence (Judd, 1992).

[22] The predetermined tolerance level for the convergence criterion 1×10^{-7}.