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EVALUATING CARBON CAPTURE AND STORAGE IN A CLIMATE MODEL WITH ENDOGENOUS TECHNICAL CHANGE

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We assess the extent to which Carbon Capture and Storage (CCS) and R&D on this abatement technology are part of a socially efficient solution to the problem of climate change. For this purpose, we extend the intertemporal model of climate and directed technical change developed by Acemoglu *et al.* (2012) [The environment and directed technical change. *American Economic Review*, 102(1), 131–166] to include a sector responsible for CCS. We show that two types of solutions exist: a renewable energy regime where current CCS technology is only temporarily used but never further developed; and a fossil energy regime where CCS is part of a long-term solution and is further developed at about the same rate as fossil energy technology. Our computations show that for current estimates of the marginal cost of CCS, the renewable energy regime clearly dominates the fossil fuel energy regime.

Keywords: Carbon capture and storage; renewable energy; fossil fuel energy; endogenous technical change; climate change.

JEL Codes: H23, O31, Q43, Q54, Q55

1. Introduction

In 2014, fossil fuels represented 82% of total primary energy supply (IEA, 2016e, p. 6) and are estimated to be responsible for 68% of global anthropogenic greenhouse gas (GHG) emissions (IEA, 2016b, p. xiii).[1] Energy demand is expected to further increase with 28% from 575 quadrillion British thermal units (QBTU) in 2015 to 736 QBTU by 2040 (EIA, 2017, p. 9–10). This increase in demand is expected to come mainly from

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[1]The 2014 shares for total primary energy supply are: coal 28.6%, oil 31.3%, natural gas 21.2%, nuclear 4.8%, hydro 2.4%, biofuels and waste 10.3%, geothermal, solar, wind, heat, etc. 1.4% (IEA, 2016e, p. 6).

non-OECD countries spurred by strong economic and population growth.[2] Without specific actions, atmospheric CO_2 concentration will continue to grow. By raising the global average surface temperature and the global mean sea level, this may prove disastrous for future generations (IPCC, 2014a). Hence the international call for decoupling global CO_2 emissions from economic growth by decarbonizing the energy system (IEA, 2016c, p. 26).

Three main policies have been proposed as possible solutions to the problem of climate change: the more intensive use of renewable and nuclear energy; the more efficient generation of power and end-use of energy; and the development and deployment of technologies to capture and store carbon emissions from fossil fuel use.[3] Carbon capture and storage (CCS) technology can be used by large stationary point sources such as fossil fuel-fired power plants and emission-intensive industrial facilities. Its main purpose is to prevent CO_2 emissions from entering the atmosphere. The rates of carbon captured can be as high as 85–95%, in both the pre- and post-combustion systems.[4] Herzog (2011) and more recently Bui *et al.* (2018), Budinis *et al.* (2018), and Durmaz (2018) provide an overview of the status of CSS as a tool in contributing to the reduction of CO_2 emissions, reflecting on four kinds of issues that must be resolved for CCS to be scaled up before this technology can become a significant contributor to the reduction of CO_2 emission: cost, transportation infrastructure, subsurface uncertainty, and regulatory and legal issues. Our paper will exclusively deal with the first issue.

The development of CCS technologies has been advocated by both national governments and international organizations. For example, some high-income oil- and gas-producing countries in Europe and North America (like Norway and Canada) are strongly committed to the use of resources in the research, development, and demonstration (RD&D) of CCS technologies. Indeed for such countries, CCS is a form of risk management response providing a hedge against the decarbonization of the energy system (IEA, 2013, p. 8).

Several international and intergovernmental agencies, including the International Energy Agency (IEA) and the Intergovernmental Panel on Climate Change (IPCC),

[2]These are the "Reference case" projections by EIA. Energy consumption in non-OECD countries is projected to increase by 41% between 2015 and 2040 in contrast to a 9% increase in OECD countries (EIA, 2017, p. 9–10).

[3]The availability of CCS technology also has important implications for bio-energy with CCS, which can offer the prospect of energy supply with large-scale net negative emissions when achieving 2°C target (Clarke *et al.*, 2014, p. 451 and IPCC, 2014b, p. 23). Another possible policy solution entering recent debate is geoengineering, the intentional, large-scale manipulation of the earth's climate system. See Rasch *et al.* (2008), Cicerone (2006), and Barrett (2008).

[4]There are three methods for capturing CO_2. *Post-combustion* carbon capture removes carbon from coal fired power generation or natural gas combined cycles after combustion. Here, CO_2 is separated from the flue gases (whose main constituent is nitrogen) using a liquid solvent. In *pre-combustion* carbon capture, fuel is pretreated and converted into a mix of CO_2 and hydrogen. The hydrogen is then separated from the carbon before being burned to produce electricity. In the *oxy-fuel combustion process*, the fuel is burned using oxygen rather than air. The result is a flue stream of CO_2 and water vapor. Because no nitrogen is present, CO_2 can be easily removed (Golombek *et al.*, 2011; Metz *et al.*, 2005). See Table 1.1 in IEA (2016a) for an overview of large-scale CCS projects in operation or under construction.

also envision an important role for CCS as part of an environmentally sustainable global energy policy, and therefore point to the need for significant R&D efforts today in order to endow the world with an economic carbon capturing and storage technology. Already in 2005, in a special report on carbon dioxide capture and storage, IPCC (2005, p. 12) estimated that CCS may have the potential to provide 15% to 55% of the world's cumulative GHG mitigation efforts up to 2100. In its later Assessment Reports, the IPCC expresses the required reliance on CCS by giving the percentage increase in mitigation costs that nondeployment of CCS would lead to under different CO_2 concentration targets. For example in Clarke *et al.* (2014, Fig. 6.24) and IPCC (2014a, Table SPM2), the range for the estimated relative increase in discounted mitigation cost for scenarios aiming at a 450 ppm CO_2 concentration in 2100 when CCS is unavailable is 29–297%, with a median estimate of 138%.[5]

In its yearly Energy Technology Perspectives (ETP) publication, the IEA presents the shares of the different mitigation technologies in the global cumulative CO_2 reductions required for the 2 degree scenario (DS) compared to the emission levels associated with a baseline scenario (e.g., 6 DS). We present the numbers in Table 1.

Thus, while the IEA attributes a significant role to CCS in the decarbonization of the global energy system, it also transpires that this role has been adjusted downwards on recent years. It is unclear whether this adjustment is due to relative cost considerations or to restrictions on the deployment of CCS technology built into the integrated assessment model to accommodate for the fact that large-scale expansion seems difficult (IEA, 2016a, p. 46–49) For example, the IEA (2009) technology roadmap projected 100 CCS plants by the year 2020 but as of 2017 there are only

Table 1. IEA estimates of shares (%) in cumulative CO_2 emission reductions (present til 2050).

	CCS	Renew. energy	Nuclear energy	Efficiency
ETP 2008 (from baseline to Blue Map)[a]	19	21	6	54
ETP 2012 (from 4°C to 2°C)[b]	20	29	8	43
ETP 2016[c]				
–From 6°C to 2°C	12	32	7	49
–From 6°C to 4°C	6	38	7	49
–From 4°C to 2°C	15	29	7	49

Notes: [a]IEA (2008, p. 38–41). Blue scenario: 50% of 2005 emissions level in 2050. [b]IEA (2012, p. 35, 36, 39). 4DS corresponds to annual energy related emissions to rise by 27% from 2009 to 2050. 2DS corresponds to annual energy-related emissions in 2050 to be at 50% of the 2009 level. IEA (2012) does not give the results for a transition from 6°C to 4°C but mentions that the 4°C scenario includes some deployment of CCS, although only 2% of total electricity capacity would be equipped with this technology in 2050. [c]IEA (2016d, p. 33).

[5]For its 5th Assessment Report, the IPCC solicited 300 baseline scenarios and 900 mitigation scenarios through an open call from integrated assessment modeling teams (31 models) (IPCC, 2014b, p. 8, fn. 12). For a list of models see Krey *et al.* (2014).

17 operational large-scale CCS projects and the rate of capture and storage would need to increase tenfold in 2025 to get on track to meet the 2DS (IEA, 2017, p. 34). Moreover, while the potential of CO_2 capture was estimated at an annual 2.6 to 4.9 $GtCO_2$, corresponding to 9–12% (IPCC, 2005, p. 24), only 30 $MtCO_2$ was captured in all CCS projects in 2017 (de Coninck *et al.*, 2018, p. 326). Nevertheless, several trajectories that are consistent with limiting global warming to 1.5° by 2100 suggest that CCS will play an important role and that over 400 $GtCO_2$ can be stored cost effectively in the power industry by 2100 (de Coninck *et al.*, 2018, p. 326).

Without doubt, these major reports have been very useful in informing both policymakers and the general public both about the threats of global warming and about the available options and costs involved when directing emissions to a more sustainable trajectory. At the same time, however, the welfare economic trade-offs underlying the results are not always transparent, i.e., it is not always clear to what extent differences in scenarios are the result of differences in the constraints imposed (emission caps, technological, and economic constraints) or the differences in trade-offs for which the models allow (e.g., between economic growth and environmental quality). Neither are these studies always clear about the assumptions made on technological progress, even though the mitigation solutions are based on complex technologies that benefit from R&D.[6]

In this paper, we wish to assess the scope for CCS and CCS R&D as part of a socially efficient solution to the climate change problem. The vehicle that we use for this purpose is the intertemporal model of climate and directed technical change developed by Acemoglu *et al.* (2012, AABH hereafter). In this model, final good production requires two inputs, fossil fuel and nonfossil fuel energy. Both types of energy are produced using labor and capital with the help of the latest available technologies. These technologies result from costly R&D efforts, and given a finite number of scientists, faster technological progress in one sector needs to be balanced against slower progress in the other sector. The production of fossil fuel energy increases the stock of CO_2 in the atmosphere, and therefore contributes to a global increase in temperature. This global warming in turn reduces the quality of the environment and with it the welfare of the representative consumer.

To this model, we append a new sector, that for CCS, which also operates using labor and capital. Like both energy sectors, the CCS technology may be improved by devoting resources to R&D. We calibrate our model using both data on world energy production levels and estimates of the marginal cost of CCS. We then ask ourselves

[6] Nordhaus and Sztorc (2013) express a related concern about the representative concentration pathways (RCPs) of the IPCC (2013): "[They] have the strong advantage of providing a coherent set of inputs for the calculations of climate and ecological models. However, the RCPs are only weakly linked back to the economic drivers of emissions. The models that produce the concentrations and forcings are based on economic and energy models. However, there is no attempt to harmonize the output, population, emissions, and other driving variables across different scenarios. Putting this differently, the IPCC RCPs have very little value in integrating the economic policies and variables with the geophysical calculations and projections." Nordhaus and Sztorc (2013, p. 23).

the following questions: (i) is it socially optimal to include CCS in today's or the near future's mitigation portfolio? and (ii) is it socially optimal to devote R&D resources to improve CCS technology, such that it becomes part of an optimal mitigation policy in the more distant future? We offer two results. Our first result is of a qualitative nature and related to the second question. It points to a nonconvexity in the model such that the optimal policy for high levels of marginal cost of CCS is qualitatively different from that for low levels. More precisely, there exists a critical level for the CCS marginal cost above which R&D resources are devoted first to dirty energy technology, later to clean energy technology, but never to the CCS technology. On the other hand, for a marginal cost below this critical level, R&D resources are devoted to both dirty energy and CSS technology, and the latter is actively used in the nearer future. For our calibrated model, the critical marginal cost of CCS entails a mark-up over the marginal cost of dirty energy between 26% and 27%;[7] this is well below the current estimates of the mark-up on the levelized cost of electricity (LCOE) — see Table 2. Our second result is that even when the marginal cost of CCS is above its critical level such that it is not optimal to further develop this technology, CCS may be optimally deployed in the nearer future, especially when global warming concerns are very strong. To sum up: for reasonable current estimates of the incremental cost of abating emissions from fossil fuel energy by means of CCS, the technology should temporarily be used to avoid too large increases in temperature, but it should not be developed further.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 details the model. In Section 4, we explain the calibration of the model. Section 5 provides our main results based on the cost mark-up of CCS in total production. Section 6 provides a rationale for the numerical results in terms of the implied elasticities of substitution between "knowledge inputs" in total productivity. In Section 7, we discuss the sensitivity of our results with respect to (w.r.t.) (i) our assumption about the elasticity of substitution between energy sectors, (ii) the level of the maximal concentration of atmospheric CO_2, and (iii) the probabilities of successful research. Section 8 concludes.

2. Related Literature

In recent years, a literature has developed that studies the desirability of CCS as part of an optimal policy to combat climate change. Such policies have been derived under various constraints (see Durmaz, 2018 for a survey). First-best (Pareto-efficient) climate policies refer to decisions about production, emission, abatement, and R&D that are only constrained by the economy's technology and balance constraints. With a sufficiently broad set of policy instruments (taxes, subsidies, etc.), such decisions can

[7]The corresponding cost per ton CO_2 avoided varies according to the deployed capture technology (see Table 2 below).

be implemented in a market economy. Second-best policies refer to the optimal use of some policy instruments (e.g., subsidies for renewable energy production, R&D subsidies) when others (e.g., a CO_2 tax) are lacking or their level cannot be changed; then the first-best decisions can no longer be implemented and efficiency losses occur. This literature on the desirability of CCS has evolved in several directions: partial versus general equilibrium models, theoretical models versus numerical solutions to empirically calibrated models, and models encompassing exogenous versus endogenous technical progress.

An early contribution to this literature is by Goulder and Mathai (2000) who develop a partial equilibrium model to answer the question about how the endogenization of technological progress affects the optimal trajectories for abatement activity and carbon taxes. They show both analytically and through numerical simulations that endogenous technical progress w.r.t. abatement activity (what they term "induced technical change" or the possibility of reducing the cost of abatement through devoting resources to R&D) in general lowers the time profile of optimal carbon taxes, and shifts at least some abatement activity from the present to the future.

Concerned with the optimal timing of CCS, Amigues *et al.* (2016) study the effects of learning-by-doing (LbD) in CCS on the social cost of carbon and energy price dynamics when all energy types are perfect substitutes. For a sufficiently high cost of solar energy, the study shows that CCS is employed once the carbon budget is spent and the carbon emissions must be zero (call this the "ceiling phase"). For a sufficiently low cost of solar energy, solar power is generated during the ceiling phase. The constant drop in the cost of CCS leads to a subsequent phase of fossil fuel energy generation with CCS. The authors show that LbD leads to nonmonotonous paths for the energy price and the use of CCS.

In a related paper, Kollenbach (2015) studies the use of CCS when there exists an exogenous ceiling on CO_2 emissions. The paper stresses the endogenous nature of the profitability of research on the clean technology: resources spent to improve that technology cannot be used for capital accumulation, nor on abatement (CCS) which allows for a higher level of fossil fuel extraction. The advantageousness of CCS depends positively on the relative cost advantage of fossil fuel energy versus the backstop technology, and negatively on the marginal abatement cost. It is shown that positive abatement may be optimal even before the CO_2 ceiling is binding because it allows for an intertemporal substitution of consumption

However, the more recent literature has often taken a general equilibrium approach. We can discern at least two separate strands in this literature. One is concerned with the characterization of socially efficient environmental policy, and its implementation in a decentralized market economy, possibly under some second-best policy restrictions (such as upper bounds on the tax rate set on carbon emissions). Examples include Grimaud and Rouge (2014) and Ayong Le Kama *et al.* (2013). The other strand compares the welfare costs of different (portfolios of) policy instruments when CO_2

stabilization or maximum temperature change targets are imposed. Examples include Gerlagh and van der Zwaan (2006), Grimaud *et al.* (2011), and Kalkuhl *et al.* (2015).

Gerlagh and van der Zwaan (2006) use a top-down computable general equilibrium model with an environment module to which they append a CCS sector. Technical progress in this sector stems from LbD. Assuming a marginal cost of abatement of \$45/ton CO_2 avoided, they compute the carbon emission trajectories for 30 five-year periods (2000–2150) under five stabilization targets (ranging from 450 to 550 ppm — particles per million) and five policy scenarios in addition to a business-as-usual scenario. Their results reveal that irrespective of the stabilization target, subsidization of renewable energy use is the most expensive policy, while a carbon emission tax of which revenue is recycled as a subsidy for nonfossil energy use represents the least costly policy mix. A carbon tax also dominates a policy that charges for fossil fuel use because it incentivizes the use of CCS activity. While CCS activity is low to begin with, about 30–50% of new fossil fuel capacity from 2050 onwards is complemented with CCS equipment.

Grimaud *et al.* (2011) extend the Goulder and Mathai (2000) framework to a general equilibrium setting. They model a decentralized market economy where energy, capital, and labor are combined into a final good. Energy is produced from nonrenewable fuels and a renewable energy source. Growth is endogenous and depends on R&D investments used to promote the efficiency of use of energy in final good production, the efficiency of producing renewable energy, or the efficiency of CCS in reducing the emissions resulting from the use of fossil fuels. In this market economy, investors are able to capture only a fraction of R&D returns and this motivates the use of (differentiated) R&D subsidies. Assuming a cap on atmospheric carbon concentration (450 or 550 ppm), they then provide a general characterization of the second-best trajectory for the tax on carbon emissions and the three R&D subsidies that maximize social welfare. In particular, the carbon tax is shown to follow an inverted U-shaped trajectory. Their main finding is that both tax and subsidy instruments should be used simultaneously to provide the strongest impact, and that R&D in CCS is warranted in the medium-term only if accompanied by the imposition of a ceiling on the stock of atmospheric CO_2.

Grimaud and Rouge (2014) also adopt a general equilibrium approach. In their model, endogenous growth is restricted to the final goods industry. Final output makes use of intermediate goods (embodying technology), labor, and the extracted amounts of a nonrenewable energy resource. The use of energy in production causes emissions that can be captured and stored using labor. With a constant and inelastic labor supply, the main trade-off in their model is between output production and abatement. The authors first characterize the socially optimal trajectories with and without access to a CCS technology, and then trace out the paths for a decentralized economy when only second-best policy tools are available. They find that the greatest abatement effort should take place in the near future, and thereafter gradually decline over time. Moreover, compared with an economy without a CCS technology, the availability of

CCS speeds up the optimal extraction rate, diverts labor from research, and in turn, lowers the output growth.

Using a dynamic general equilibrium model, Kalkuhl *et al.* (2015) study the implications of various second-best policies (such as, a CCS (renewable energy) policy that only subsidizes the fossil fuel and CCS (renewable energy) sector, and a hybrid policy that subsidize both) for the welfare. CCS policies lead to relatively fewer emissions initially and less drastic reductions later on. Conversely, renewable energy policies lead to higher emissions earlier and strong declines later on. Under the CCS policy, the CCS activity increases significantly until 2050 after which fossil fuels starts to be replaced by renewables due to the increasing extraction costs and scarcity and, with it, a decreasing price of renewable energy relative to the fossil fuel price.

Finally, in a static multi-market general equilibrium model for Europe, Golombek *et al.* (2011) look at the development of CCS in relation to technology-neutral abatement policies (i.e., carbon taxes or tradable permits).[8] When an uniform tax of \$90/tCO_2 is implemented, the results show that new coal power plants with CCS become profitable, totally replace non-CCS coal power investments, and partially replace new wind and biomass power plants. For the same tax level, new gas power plants with CCS become profitable and replace almost all non-CCS power investments. Compared to a business-as-usual scenario, this leads to a 90% lower CO_2 emissions in 2030. The results also imply that from a social point of view it is not desirable to retrofit CCS into the existing coal and gas power plants.

Our model shares several aspects with the models just described. We employ a global and dynamic general equilibrium setting with four sectors: a "dirty" fossil fuel energy sector, a "clean" nonfossil fuel energy sector, a CCS sector, and a sector transforming clean and dirty energy into a final good that is used for consumption and capital investment. In addition to the standard labor balance constraint, the economy is endowed with a stock of scientists who can be allocated to each of the three lower-level sectors (clean, dirty, and CCS) where their efforts result in efficiency-enhancing innovations. Our model, therefore, builds on the recent literature on directed technical change (see Acemoglu, 2002, 2003, and AABH).[9] Moreover, rather than imposing exogenous stabilization targets, we let the quality of the environment enter consumer welfare (cf. Tahvonen and Kuuluvainen, 1991; Bovenberg and Smulders, 1995, and Grimaud and Rouge, 2014). We are primarily interested in whether CCS activity and CCS-related R&D effort are part of a first-best policy. We characterize the socially optimal solution, proceed by a numerical calibration of our model in the same vein as

[8]Equilibrium is calculated for exogenously taken non-EU parameter values.

[9]An early example of directed technical change in the analysis of optimal environmental regulation is Bovenberg and Smulders (1995). Bovenberg and Smulders (1996) extend this paper by studying the transitional dynamics. Other examples are Goulder and Schneider (1999), van der Zwaan *et al.* (2002), Popp (2004), Popp (2006), Grimaud and Rouge (2008), and Gerlagh *et al.* (2014).

AABH, and then optimize as in Gerlagh and van der Zwaan (2006) over a finite but long discrete horizon (30 10-year periods).

3. The Model

The AABH general equilibrium model consists of three sectors: a clean and dirty energy sector, and a final goods sector. We augment that model with a fourth sector responsible for CCS activity. We are primarily interested in the Pareto-efficient policy allocation of resources. With a sufficiently large set of instruments (taxes, subsidies to R&D activities), this allocation can be implemented in a market economy.

An infinitely lived representative consumer cares about a final good (c_t) and the quality of the environment (F_t) in each period t of life. The period utility function, $U(c_t, F_t)$, satisfies the standard monotonicity and concavity assumptions. The final good (Y_t) is produced by means of two energy types: dirty energy (Y_{dt}) and clean energy (Y_{ct}). The (symmetric) production function is assumed to display a constant elasticity of substitution (CES), ε

$$Y_t = \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

Each of the two energy types j ($j = c, d$) is produced using labor (L_j) and capital (or machinery, x_j). There is a fixed amount of workers available in every period (normalized to 1). Capital is available at a constant marginal cost ψ, and assumed to fully depreciate after one period.[10] The sector production functions are of Cobb–Douglas (CD) form, with technology parameter (sectoral stock of knowledge) A_j ($j = c, d$). The same is true for the CCS sector, which we label with index a (for abatement). Thus

$$Y_{jt} = A_{jt}^{1-\alpha} L_{jt}^{1-\alpha} x_{jt}^{\alpha} \quad (j = a, c, d). \tag{2}$$

The stock of knowledge/technology level A_{jt} in sector j is assumed to grow at a rate of $\gamma\eta_j s_{jt}$, where s_j is the number of researchers allocated to sector j ($j = a, c, d$), η_j is the probability that a single researcher is successful in creating an innovation, and γ is the relative increase in knowledge in the case of such an innovation.[11] Subsequently, A_j evolves according to

$$A_{jt} = (1 + \gamma\eta_j s_{jt})A_{jt-1}. \tag{3}$$

[10] Similar to Golosov *et al.* (2014), when assuming full depreciation, we have in mind a time period of at least 10 years. In our numerical simulations, a period will accordingly constitute 10 years.

[11] Thus, there are constant returns to scale (CRS) in research. However, arguments that may provide deviations from CRS in both directions exist. For instance, “fishing out” problems, where easy inventions occur sooner with little effort whereas larger technological challenges are solved later and require more effort, indicate decreasing returns to scale, while positive spillovers between researchers and/or labs suggest increasing returns to scale. See Mattauch *et al.* (2015) for a variant of the AABH model with technical progress stemming from LbD.

In each period, a fixed amount of scientists (which we normalize to 1) is available and the allocation of a scientist to one sector fully crowds out R&D activity in the other sector/s with the same amount. Thus scientists are not part of the regular labor force.[12]

With an activity level Y_a in the CCS sector, the emissions corresponding to Y_a units of dirty energy input production are captured and stored. The atmospheric concentration of CO_2, S, then evolves according to the following equation of motion:

$$S_t - S_{t-1} = \xi(Y_{dt-1} - Y_{at-1}) - \delta(S_{\text{dis}} - S_{t-1}), \tag{4}$$

where ξ is the rate of CO_2 emissions from dirty energy production, δ is the decay rate of the CO_2 stock in the atmosphere, and S_{dis} is the disaster carbon concentration.[13] The degree of atmospheric carbon concentration translates into an environmental quality index $\tilde{F}(S_t)$ with range [0,1] and $\tilde{F}' < 0$ (see p. 15 in Section 4). One interpretation of $\tilde{F}$ is the fraction of final good left for consumption that remains after the damages due to global warming are subtracted, which would warrant the specification $U(c_t, \tilde{F}_t) = u(c_t\tilde{F}_t)$.[14] However, below we will also allow for a more general interaction between final good output and environmental quality.

As we are primarily interested in the Pareto efficient policy, we consider the levels of the labor and capital inputs, the level of energy production, the level of CCS activity, and the allocation of scientists that maximize the intertemporal utility of the representative consumer subject to the technology constraints, the equation of motion for the environment and for the sectoral stocks of knowledge, and the balance constraints

[12] Pottier *et al.* (2014) criticize the AABH model for the assumption of an exogenously given number of scientists. Nevertheless, AABH and our model builds on the recent literature on endogenous technical change (see Acemoglu, 2002, 2003). In this regard, the assumption that researchers are scarce, and thus, that there is full crowding out between the different forms of R&D, is standard in this literature. As the number of researchers is constant, economic growth cannot be sustained by increasing the amount of these factors. To maintain growth, there is state dependence in the innovation possibilities frontier. In this regard, the spillovers from previously accumulated knowledge in one sector make researchers in that sector more productive over time. Furthermore, we assume that the aggregate innovation production function has constant returns to accumulated knowledge. This is because if the returns to the accumulated knowledge are slightly higher, the model generates explosive growth. On the other hand, if there are decreasing returns to the accumulated knowledge, productivity growth gradually ceases. The assumption that the number of researchers is constant also allows us to avoid any scale effect on output growth (Jones *et al.*, 1999; Groth, 2007). For example, if the number of researchers was subject to exponential growth, the growth rate of the output in our model would itself grow exponentially.

[13] Thus the quality of the environment is defined as $S_{\text{dis}} - S_{t-1}$ which drives regeneration. With a lower bound on the atmospheric concentration of CO_2 corresponding to the pre-industrial level ($\underline{S}$), the equation of motion becomes: $S_t = \min\{\underline{S}, \xi(Y_{dt-1} - Y_{at-1}) + S_{t-1} - \delta(S_{\text{dis}} - S_{t-1})\}$. Absent emission activity, the lower bound would be approached at a constant natural decay rate δ. Alternatively, atmospheric concentration could be assumed to decay at a rate depending positively on the discrepancy between S_{t-1} and $\underline{S}$, viz. $S_t - S_{t-1} = -\delta(S_{t-1} - \underline{S}) + \xi(Y_{dt-1} - Y_{at-1})$. Absent emissions, the gap between S_t and $\underline{S}$ narrows down at a decreasing rate and converges to zero asymptotically. See, e.g., Bovenberg and Smulders (1995, 1996). In our simulations, the lower bound is never binding. Hence, we ignore it in the remainder of this section.

[14] In this respect, the AABH model and our extension differ from the models of Nordhaus and Sztorc (2013), Nordhaus (2018) and Golosov *et al.* (2014) where the net-of-damage function $\tilde{F}$ is multiplied with total *production*. Thus in the latter models the increase in atmospheric CO_2 concentration reduces what is available for both consumption and investment. In the former models, only consumption is affected.

for labor and scientists

$$\max_{\{Y_t, Y_{jt}, L_{jt}, x_{jt}, A_{jt}, s_{jt}\}_{t=0,\dots,\infty}^{j=c,d,a}} \sum_{t=0}^{\infty} \beta^t U(Y_t - \psi(x_{ct} + x_{dt} + x_{at}), \tilde{F}(S_t)),$$

$$\text{s.t } Y_t = \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (\pi_t),$$

$$Y_{jt} = A_{jt}^{1-\alpha} L_{jt}^{1-\alpha} x_{jt}^{\alpha} \quad (\pi_{jt}) \quad (j = a, c, d),$$

$$A_{jt} = (1 + \gamma\eta_j s_{jt}) A_{jt-1} \quad (\mu_{jt}) \quad (j = a, c, d),$$

$$S_t - S_{t-1} = \xi(Y_{dt-1} - Y_{at-1}) - \delta(S_{dis} - S_{t-1}) \quad (\omega_t),$$

$$L_{ct} + L_{dt} + L_{at} \leq 1 \quad (w_t),$$

$$s_{ct} + s_{dt} + s_{at} \leq 1 \quad (v_t),$$

$$Y_{at} \leq Y_{dt} \quad (\phi_t).$$

In this problem, β is the discount factor, ψ is the amount of final goods necessary to build a machine, and the Lagrange multipliers in brackets following the constraints are all current values (thus the net present value of a marginal unit of labor in period t is $\beta^t w_t$). The final inequality precludes the more than 100% capture of CO_2 emissions (we ignore the fact that existing CCS technology does not allow for capture rates exceeding approximately 90%). In a market economy, these decisions can be decentralized by means of a tax on emissions, subsidies to research on clean energy production and CCS technology, subsidies to machine use (to correct for possible market power of machine producers), and lump-sum transfers to the representative consumer. The interested reader is referred to the AABH paper.[15]

The social marginal environmental damage of period t emissions is given by $\beta\xi\frac{\omega_{t+1}}{\pi_t}$. Here, ω_t, the shadow value of the environment at time t, is the discounted intertemporal sum of marginal disutilities caused by the current dirty input production, which is adjusted for the regeneration in every period

$$\omega_t = \sum_{k=0}^{\infty} \beta^k (1 + \delta)^k U_{F_{t+k}}(-\tilde{F}'_{t+k}).$$

Thus the tax that emitters should face in a decentralized economy to make them aware of the social damages is given by $\beta\xi\frac{\omega_{t+1}}{\pi_t}$.

In the sequel, we define the social price of sector j output as $\widehat{p}_{jt} \stackrel{\text{def}}{=} \frac{\pi_{jt}}{\pi_t}$ $(j = c, d, a)$. For our purposes, it will be useful to express the tax rate in a slightly different way, viz., as a fraction of the social marginal damage of dirty energy: $\tau_t \stackrel{\text{def}}{=} \xi\beta\frac{\omega_{t+1}}{\pi_t}\frac{1}{\widehat{p}_{dt}}$; it can be thought of as the *ad valorem* rate on fossil fuel energy use.

In the remainder of this section, we focus on characterizing the optimal policy w.r.t. the CCS sector, in both its level of activity and the efforts directed to R&D. We relegate the solution of the full model to Appendix A.

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[15]See also Greaker *et al.* (2018) for a discussion of the robustness of R&D subsidy policies prescribed by the AABH model w.r.t. the assumptions on the length of the patent period.

For both energy sectors, the marginal product in final good production should equal the social price: $MP_{ct} = \widehat{p}_{ct}$ and $MP_{dt} = (1 + \tau_t)\,\widehat{p}_{dt}$. For abatement activity (i.e., CCS), the optimality condition is

$$\widehat{p}_{at} \geq \tau_t \widehat{p}_{dt} - \frac{\phi_t}{\pi_t},$$

with equality whenever $Y_{at} > 0$. The second-term on the right-hand side (RHS) is the period t social cost of not being able to capture more CO_2 than the amount emitted by the dirty sector in period t; this cost is obviously zero when $Y_{at} < Y_{dt}$. Thus, when $\widehat{p}_{at} > \tau_t \widehat{p}_{dt}$, any abatement is suboptimal. If partial abatement is optimal, then $\widehat{p}_{at} = \tau_t \widehat{p}_{dt}$, while full abatement requires that $\widehat{p}_{at} \leq \tau_t \widehat{p}_{dt}$.

In Appendix A, we show that allocating a scientist to the R&D department of sector j yields a marginal social value of

$$\frac{\mu_{jt}}{\pi_t} \gamma \eta_j A_{jt-1} = \frac{1}{\pi_t} \frac{\gamma \eta_j}{1 + \gamma \eta_j s_{jt}} (1 - \alpha) \sum_{k=0}^{\infty} \beta^k \pi_{t+k} \hat{p}_{jt+k} Y_{jt+k}. \tag{5}$$

This value positively depends on (i) the productivity of R&D ($\gamma \eta_j$) and (ii) the discounted social value of the output stream ($\widehat{p}_{jt+k} Y_{jt+k}$, $k = 0, 1, \ldots, \infty$) of sector j. If R&D in sector j is optimal, then this marginal social value should match the social wage of the scientists, $\frac{v_t}{\pi_t}$. If (5) falls short of $\frac{v_t}{\pi_t}$, then R&D is not optimal in sector j. It is therefore clear from (5) that substantial CO_2 capture and storage in the near future is a prerequisite for justifying R&D in the CCS sector.[16]

The allocation of labor and capital across sectors should satisfy the standard conditions of equality between the marginal products and the corresponding social prices. In Appendix A, we show how the first-order conditions (FOC) together with the constraints allow us to reduce the above maximization problem to a simpler model in terms of four sets of decision variables: $\{Y_{at}, \tau_t, s_{ct}, s_{dt}\}_{t=0,1,\ldots,\infty}$. This problem is then calibrated and solved (with MATLAB) for a large but finite time horizon. In Section 4, we explain how the calibration is done. The optimal solutions are presented in Section 5 and discussed in Section 6.

4. Numerical Implementation of the Model

To implement the model numerically, we proceed as in AABH. We consider a long but finite horizon (300 years) and let a single period consist of 10 years.[17] We fix α to $\frac{1}{3}$.[18]

[16] In a decentralized equilibrium, this would translate into a high price and/or market size effect for CCS.

[17] In AABH a single period consists of five years. Because our model has two extra sequences of decision variables (Y_{at} and s_{at}, $t = 1, \ldots, 300$), we increase the length of a period to keep the total number of decision variables in the numerical optimization within limits. As in AABH, we take the base period ($t = 0$) as 2002–2006. Because a single period consist of 10 years in our study, we double the initial values when calibrating the model (see Appendix B). The final period ($T = 30$) is 2297–2306.

[18] This assumption ensures that the wage bill in a laissez-faire economy is $\frac{2}{3}$ of GDP.

Furthermore, we take ε, the value of the elasticity of substitution between the clean and dirty energy sectors, as 3 (van der Zwaan *et al.*, 2002; Gerlagh and van der Zwaan, 2003, 2004; Acemoglu *et al.*, 2012).[19] As each period corresponds to 10-year intervals, what is relevant for our study is the long-run elasticity of substitution. In this regard, $\varepsilon = 3$ can be seen a reasonable value.[20] In Section 7, we discuss the sensitivity of our results to the value of ε.

We calibrate the model by assuming that in period 0 (the base period) there is no environmental policy. Under this assumption, and using the values for world primary energy production by sector (Y_{d0} and Y_{c0}), we solve for the base period technology efficiency parameters A_{d0} and A_{c0}, as well as their CES average $B_0 \overset{\text{def}}{=} (A_{c0}^{-\varphi} + A_{d0}^{-\varphi})^{-\frac{1}{\varphi}}$ (with $\varphi \overset{\text{def}}{=} (1-\varepsilon)(1-\alpha) = -\frac{4}{3}$): $A_{d0} = 2658$, $A_{c0} = 1072$, and $B_0 = 3232$ (see Appendix B). Then, using the result that $MC_{j0} = \widehat{p}_{j0} = (\frac{B_0}{A_{j0}})^{1-\alpha}$ (cf. Eq. (A.16) in Appendix A), where MC_{j0} is the social marginal cost of sector j output, we obtain

$$\text{MC}_{d0} = 1.14\frac{\text{UON}}{\text{QBTU}} \quad \text{and} \quad \text{MC}_{c0} = 2.09\frac{\text{UON}}{\text{QBTU}},$$

where UON stands for *units of the numeraire* and QBTU are quadrillions (10^{15}) of British Thermal Units.

There exists a variety of estimates for the average cost of CCS, each surrounded by a wide confidence interval. In Table 2 we report average values for the LCOE at power plants with and without capture, differentiated along four capture technologies. All absolute figures are either in 2010- or 2013-USD values. The cost estimates only pertain to carbon capture and exclude the cost of transportation and storage of CO_2. These items will increase the final cost, unless the CO_2 can be stored in oil reservoirs and in that way enhance the extraction of the remaining oil. The benefits of such enhanced oil recovery (EOR) then come as a credit and may lower the final cost of CCS. The third row gives the relative increase in LCOE due to CO_2 capture and can be thought of as the mark-up of MC_a over MC_d (Rubin *et al.*, 2015).

Depending on the capture technology, the mark-up varies from 72% down to 33%. We will therefore assume a reference mark-up of 60% and carry out a sensitivity analysis for values as low as 26%. Hence, the reference cost of abating CO_2 when producing one QBTU of dirty energy is

[19]For a CES technology, the implied conditional own elasticity for the dirty energy input factor is given by ε times the cost share of clean energy factor. With $\varepsilon = 3$, the implied equilibrium prices for both energy factors (to be computed below) result in a "clean" cost share of 0.23. This means that the conditional own elasticity for dirty energy is about -0.69, which is a reasonable value. In a recent study, Papageorgiou *et al.* (2017) estimate the elasticity of substitution between clean and dirty energy between 1.7 and 2.8 for the energy sector (panel of cross-country sectoral data). Moreover, Popp (2006) considers elasticities of substitution that range from 1.6 to 8.7. For a discussion regarding the CES function and the value of the elasticity of substitution between the two energy sectors, we refer the reader to Gerlagh and van der Zwaan (2004).

[20]This value also implies that the isoquants are tangent with the input axes, but at the same time have endpoints at $Y^{\frac{3}{2}}$. Thus, although the CES specification makes it technically feasible to rely solely on nonfossil fuel energy, such a solution will not be selected as long as the (social) relative price of fossil fuel energy is finite.

Table 2. Summary of cost estimates of different types of CO_2 capture.

	SCPC[a]			NGCC[b]			IGCC[c]			Oxy-SCPC[d]		
	SRCCS[e]	RDH[f]	Fink[g]	SRCCS	RDH	Fink	SRCCS	RDH	Fink	SRCSS	RDH	Fink
LCOE w/o capture (USD/MWh)	76	70	66	55	64	77	80	90	75	—	64	62
LCOE w/capture (USD/MWh)	119	113	107	81	92	102	106	120	104	—	110	102
ΔLCOE(%)	56	62	62	52	45	33	33	34	39	—	72	64
Cost of CO_2 avoided (USD/$t$$CO_2$)	67	63	59	83	87	80	39	46	43	—	62	52

Notes: [a]Pre-combustion capture at super critical pulverized coal (SCPC) power plants using bit coal. [b]Post-combustion capture at new natural gas combined cycle power plants. [c]Pre-combustion capture at coal-based integrated gasification combined cycle power plant. [d]Oxy-combustion capture at new SCPC/USC plant using sub-bit or bit coals. [e]Special Report on Carbon Dioxide Capture and Storage (IPCC, 2005) but with the original 2002-USD values transformed into 2013-USD as reported in Rubin *et al.* (2015). [f]Figures in 2013-USD based on more recent cost compilation by Rubin *et al.* (2015). [g]Figures in 2010-USD reported in Finkenrath (2011). All figures *exclude* the cost of transportation and storage and the possibly negative cost effect of EOR.

$$\mathrm{MC}_{a0} = 0.60 \times 1.14 \frac{\mathrm{UON}}{\mathrm{QBTU}} = 0.684 \frac{\mathrm{UON}}{\mathrm{QBTU}},$$

(and thus the marginal cost of one QBTU of "cleaned" dirty energy is $1.60 \times 1.14 \frac{\mathrm{UON}}{\mathrm{QBTU}} = 1.824 \frac{\mathrm{UON}}{\mathrm{QBTU}}$).

Having found MC_{a0}, we calibrate A_{a0} using the relationship $\mathrm{MC}_a = (\frac{B_0}{A_{a0}})^{1-\alpha}$ (cf. Eq. (A.16) in Appendix A)

$$A_{a0} = \frac{B_0}{(0.684)^{\frac{3}{2}}} = 5713.$$

The quality of the environment, $\tilde{F}(S_t)$, is modeled as a decreasing and strictly concave function of the rise in temperature since pre-industrial times: $\tilde{F}(S_t) = F(\Delta t(S_t))$, where

$$F(\Delta t) = \frac{(\Delta t_{\mathrm{dis}} - \Delta t)^{\lambda} - \lambda \Delta t_{\mathrm{dis}}^{\lambda-1}(\Delta t_{\mathrm{dis}} - \Delta t)}{(1-\lambda)\Delta t_{\mathrm{dis}}^{\lambda}}.$$

Here, Δt_{dis} is the increase in temperature leading to environmental disaster. Thus $F(\Delta t)$ is an index of environmental quality with λ measuring the sensitivity to the temperature increase; it has the properties that $F(0) = 1$ and $F(\Delta t_{\mathrm{dis}}) = 0$. For $\Delta t_{\mathrm{dis}} = 6°\mathrm{C}$ and $\lambda = 0.1442$ (see AABH), the function is depicted as the solid line in Fig. 1. A λ-value of 0.1442 amounts to a 1% reduction in environmental quality following a 2°C global temperature rise. But we will also consider a more pessimistic scenario with λ-value of 0.3011; this produces 2% damage at the same temperature increase (cf. Weitzman (2010); see the dashed line in Fig. 1).

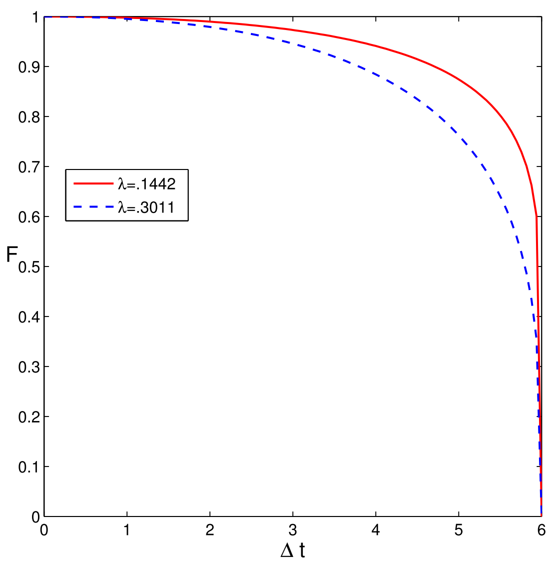


Figure 1. Damage function

The rise in temperature is an increasing function of the stock of CO_2 in the atmosphere, S_t, measured in ppm

$$\Delta t = 3\log\left(\frac{S_t}{280}\right) \Big/ \log(2),$$

where 280 refers to the atmospheric concentration of CO_2, measured in ppm in pre-industrial times. Thus a doubling of concentration increases temperature with 3°C since pre-industrial times.

World CO_2 emissions from energy consumption during the base period were 272 (2×136) billion tons (EIA 2008, Table 11.19). As $Y_{d0} = 3786(= 2 \times 1893)$ QBTU, this means an emission rate of

$$\frac{272}{3786}\,\frac{\text{billion ton } CO_2}{\text{QBTU}} = 71.85\,\frac{\text{million ton } CO_2}{\text{QBTU}}.$$

As 7.78 billion tons of emitted CO_2 give rise to an increase in atmospheric concentration of CO_2 of one ppm, the emission rate expressed as ppm per QBTU is

$$\xi = 71.85\,\frac{\text{million ton } CO_2}{\text{QBTU}} \times \frac{1}{7.78\,\frac{\text{billion ton } CO_2}{\text{ppm}}} = 0.0092\,\frac{\text{ppm}}{\text{QBTU}}.$$

The environmental quality in the base period was defined as the difference between the CO_2 concentration producing the disaster temperature rise, $2^{\frac{\Delta t_{\text{dis}}}{3}}280$ ppm, and the concentration in the base period $S_0 = 379$ ppm. With $\Delta t_{\text{dis}} = 6$, this is 741 ppm. Then δ, the regeneration rate of the environment, is set at 50% of share of emissions in the base period ($\frac{272}{7.78} = 35$ ppm) in the atmospheric quality (741 ppm), i.e., $\delta = \frac{1}{2} \times \frac{35\text{ppm}}{741\text{ppm}} = 0.0236(= 2 \times 0.0118)$ (cf. AABH). The utility function is assumed to take the CD form $U(c, F) = \frac{[c \cdot F]^{1-\sigma}}{1-\sigma}$, with $\sigma = 2$. However, because the use of this particular utility function has been criticized for allowing too easy substitution of consumption for environmental quality (Weitzman, 2010), we also ran simulations using a CES utility function with a substitution elasticity of $\frac{1}{2}$ (cf. Sterner and Persson, 2008): $U(c, F) = \frac{1}{1-\sigma}\left(\frac{1}{2}c^{\frac{\theta-1}{\theta}} + \frac{1}{2}F^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}(1-\sigma)}$ with $\sigma = 2$ and $\theta = \frac{1}{2}$. As we show below, this does not change the qualitative nature of our results (cf. Fig. 5).

Finally, we follow AABH by assuming a discount rate of 0.015 (so that $\beta = 0.9852$). The technological progress parameters are chosen as follows: $\gamma = 1$, and $\eta \equiv \eta_c = \eta_d = \eta_a = 0.2$ per 10-year period (i.e., 2% per year, cf. Jones, 2016, Table 2).

Following the numerical simulations, we carry out sensitivity analyzes. First, we consider disaster level of temperature rise of 4°C and 2°C which correspond to maximal concentrations of atmospheric CO_2 of about 700 and 450 ppm, respectively, that need to be avoided. Second, we allow for a 3% per year probability of successful research in the nonfossil fuel energy and CCS sectors (the "infant" sectors) for the first 50 years. This concludes the calibration of our model.

5. Main Results

We first present the results for $\mathrm{MC}_a = 0.684$ (recall that this corresponds to a mark-up of 60% for the levelized cost of "dirty" electricity) when preferences are of CD form and $\lambda = 0.1442$. The results are presented in Fig. 2. Panel (b) shows the time path for the optimal tax rate τ_t as well as the cost of CCS relative to the marginal cost of dirty energy. Since $\mathrm{MC}_{at}/\mathrm{MC}_{dt}\,(=\widehat{p}_{at}/\widehat{p}_{dt})$ always exceeds τ_t, it is never optimal to have capture and storage of CO_2 emissions (panel (d)). Because CCS is never active, no

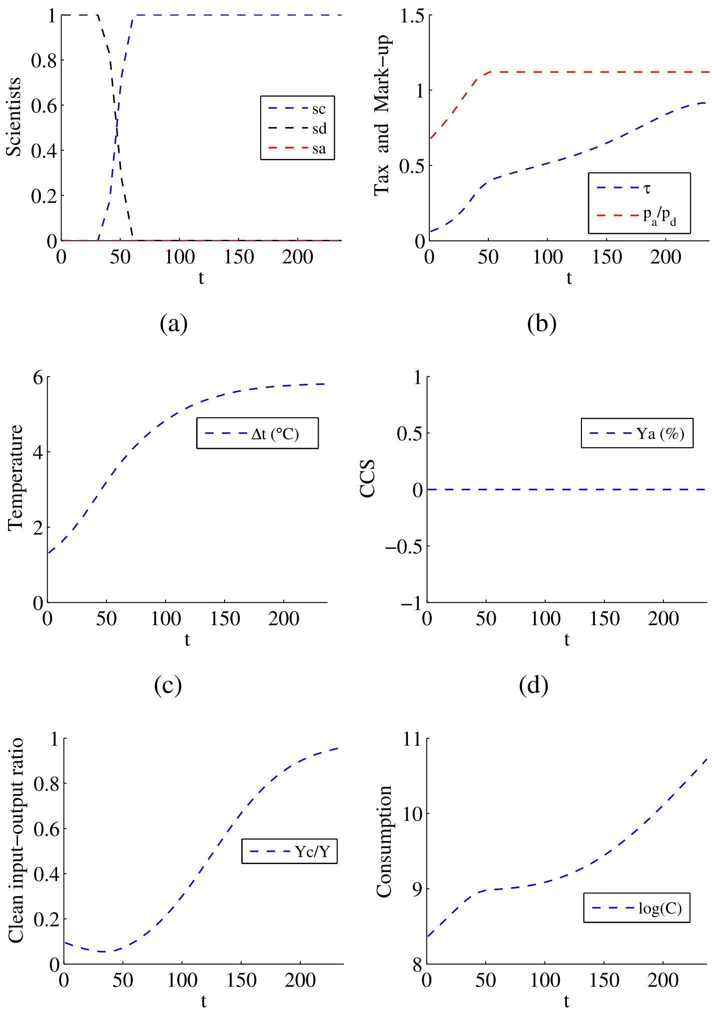


Figure 2. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = \widehat{p}_{a0} = 0.684$ (60% mark-up over MC_{d0})

scientists are allocated to CCS R&D (panel (a)), and vice versa: the absence of R&D on CCS technology means that this technology remains too expensive in use. Note that the initial R&D activity in the "dirty" energy sector increases the cost of CCS relative to that of Y_d. These trajectories are identical to those depicted in Fig. 1 in AABH. In particular, after about 50 years, scientists are relocated from the dirty energy sector in favor of the clean energy sector. Together with the tax on dirty energy, the result is a gradual increase in the intensity of clean energy in final good production (panel (e)). The temperature continues to increase but stabilizes below the disaster level of a temperature rise of 6°C. If λ is increased to 0.3011, although deteriorating the environmental impact of a (smaller-than-disaster-level) temperature rise, the overall picture remains almost the same, except that the switch from "clean" to "dirty" R&D takes place a few years earlier. The result is a slightly lower temperature increase to which the climate converges (figures available upon request).

As the current estimates for the marginal cost of CCS make neither CCS nor R&D on CCS part of the optimal policy portfolio, we ask by how much this marginal cost must fall before CCS and/or R&D on CCS start to be desirable. When $\mathrm{MC}_a = 0.55$ (corresponding to a mark-up of 48%), the use of CCS becomes optimal in the distant future. The reason is the steady increase in the tax rate on Y_d, passing the relative cost of abatement around $t = 220$. From then on, CCS becomes active, but not for long as the use of the dirty input becomes quite minimal. See Fig. 3 for details. However, in line with our explanation following Eq. (5), the fact that CCS is only active in the distant future makes it suboptimal to divert any R&D resources to that sector in the near future. We categorize these scenarios — without any R&D on CCS, but possibly with active CCS — under *Regime* 1. In Regime 1, sole reliance on clean energy is the long-run solution. Our simulations show that Regime 1 continues to hold for MC_a values as low as 0.31 (corresponding to a mark-up of 27%) — see the dashed lines in Fig. 4.[21]

For lower MC_a values, a second regime, *Regime* 2, becomes optimal. This is shown in Fig. 4 by the solid lines, which show the optimal policy for $\mathrm{MC}_a = 0.30$.[22] $\mathrm{MC}_a = 0.30$ corresponds to a mark-up of 26% for the levelized cost of "dirty" electricity. In this regime, CCS becomes active after 50 years, i.e., sooner than in Regime 1, and fossil fuel energy generation without CCS phases out almost entirely in 150 years (see panel (d)). The second and main difference w.r.t. Regime 1 is that R&D in CCS now becomes part of the optimal research policy (see panel (a)). Whereas in Regime 1 only clean R&D prevails in the distant future, there is no role at all for "clean" R&D in Regime 2. Hence, the relative inefficiency of "clean" technology becomes permanent.

[21]At $\mathrm{MC}_a = 0.31$, CCS becomes active after 70 years and activity increases to as high as 100% around $t = 220$, after which it begins to decline. But these high CCS rates do not necessarily imply a growth in the absolute amounts captured and stored. The reason is the diminishing use of the dirty energy in Regime 1, as shown in the lower left panel of the figure.

[22]In order not to have many line styles, we use coloring to distinguish different variables within scenarios and different line styles (solid versus dashed) to distinguish the same variables across regimes.

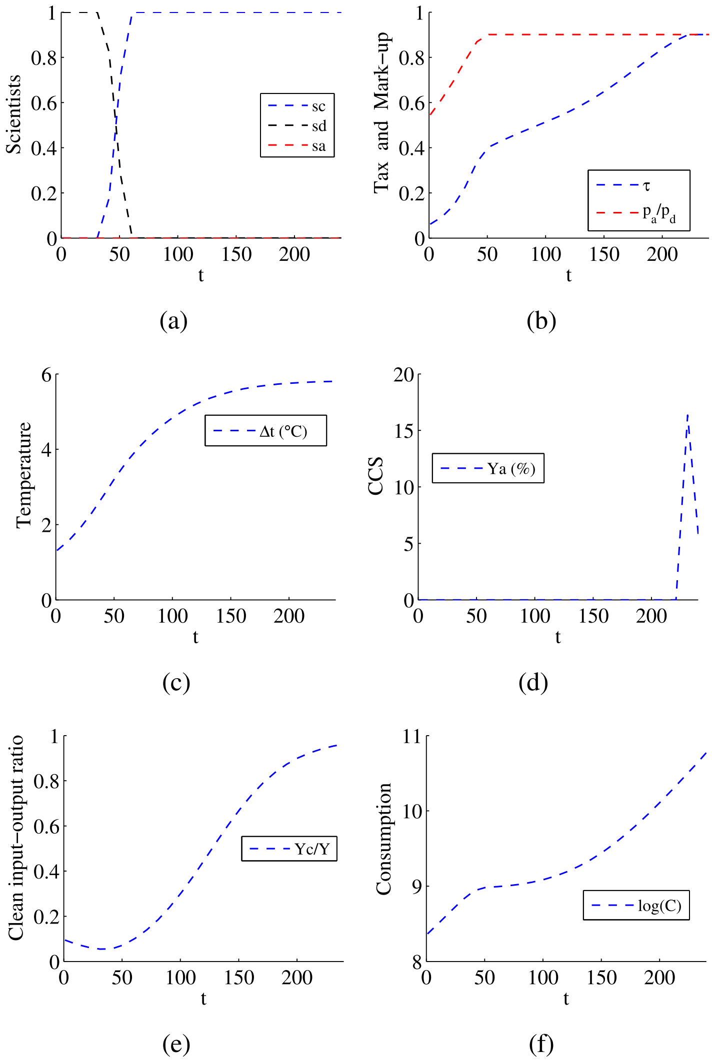


Figure 3. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = 0.55$ (48% mark-up over MC_{d0})

Conversely, "dirty" R&D dominates for about 100 years, after which the scientists are shared about equally with the CCS sector. Thus in Regime 2, the economy relies in the long run only on cleaned fossil fuel energy.

In Fig. 5 we have plotted the maximal intertemporal welfare against values for MC_{a0}. Recall that $\mathrm{MC}_{a0} = 0.684(0.25)$ corresponds to a mark-up of 60%(22%) over MC_{d0}. The maximal value function is drawn for the scenario described above (CD preferences, $\lambda = 0.1442$) but also for the case of CES preferences and $\lambda = 0.3011$. For all scenarios, we discern the same pattern: Regime 1 for modest to high CCS marginal cost values and Regime 2 for sufficiently low values.

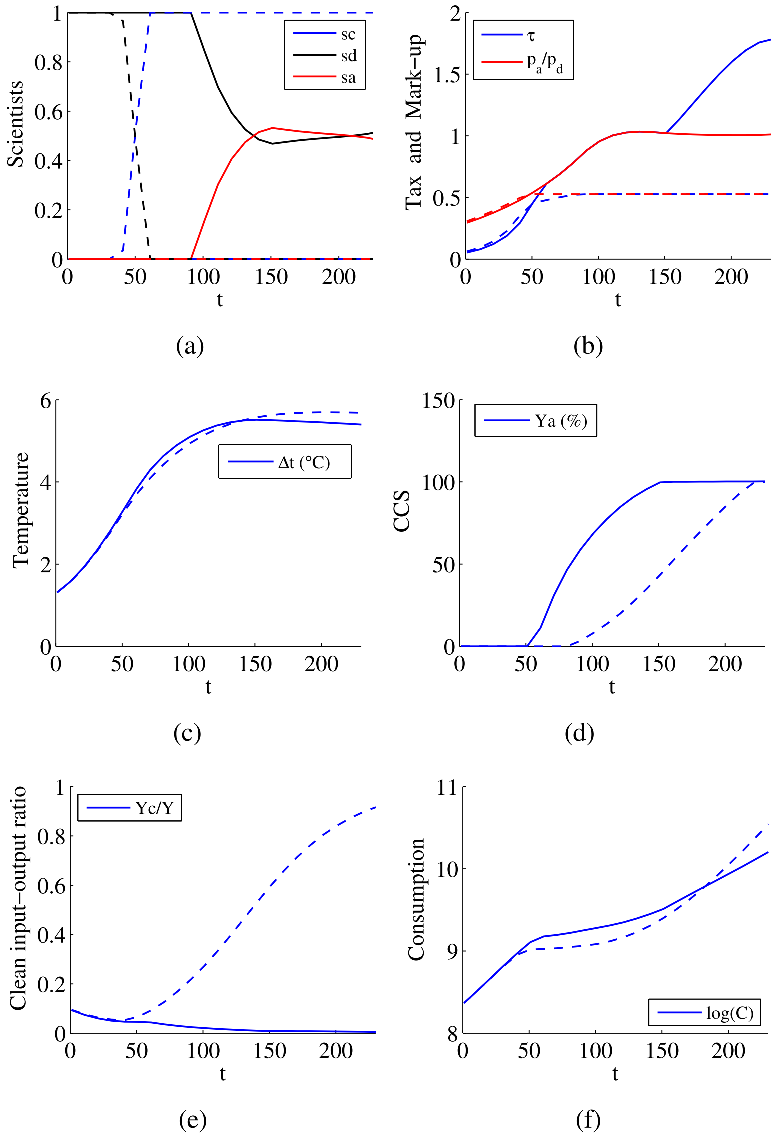


Figure 4. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = 0.3$ (26% mark-up over MC_{d0}) (solid lines refer to Regime 2 and dashed lines refer to Regime 1)

The switch from Regime 1 to Regime 2 when MC_a drops below some critical value in [0.30, 0.31] points to a nonconvexity in the model owing to the endogenous nature of the R&D activity. We will elaborate on this in Section 6.

6. Discussion

Before moving to the sensitivity analysis, it is useful to interpret the results obtained so far. The main objectives of the planner is to secure long-run consumption growth and a

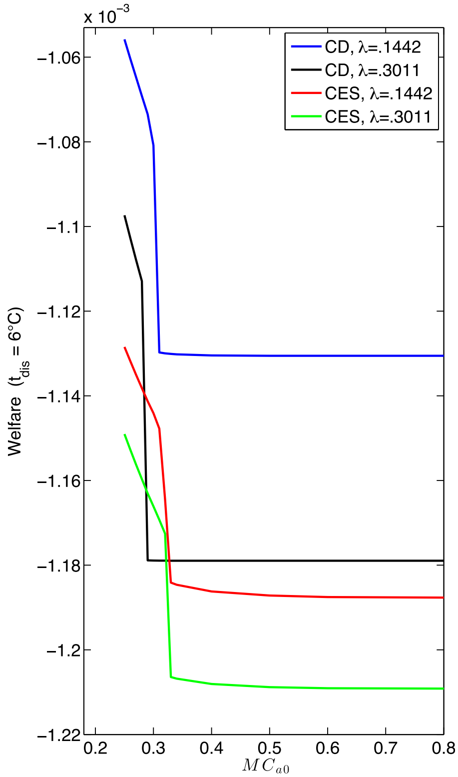


Figure 5. Maximal intertemporal welfare levels given MC_{a0}

sustainable quality of the environment. The latter objective can be achieved either through an intensive use of nonfossil fuel energy or by capturing the CO_2 emissions from fossil fuel energy production. The first alternative is initially about 14.6% more expensive than the second ($MC_{c0} = 2.09, MC_{d0} + MC_{a0} = 1.824$). Consumption growth depends on output growth, which, in turn, depends on overall productivity growth. The overall productivity index for the economy is a weighted average of the efficiency parameters A_j ($j = c, d, a$). In Appendix A, Eq. (A.15), we show that this index is given by

$$B_t \stackrel{\text{def}}{=} \left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi} \left[1 + \min \left\{ \left(\frac{A_{dt}}{A_{at}} \right)^{1-\alpha}, \tau_t \right\} \right]^{1-\varepsilon} \right)^{-\frac{1}{\varphi}}.$$

In order to sketch a clear picture, let us abstract from situations with partial abatement. In Regime 1, when CCS is absent, and thus, $\tau_t < (A_{dt}/A_{at})^{1-\alpha}$, the productivity index simplifies to

$$B_t = (A_{ct}^{-\varphi} + A_{dt}^{-\varphi}[1 + \tau_t]^{1-\varepsilon})^{-\frac{1}{\varphi}}. \tag{6}$$

We can think of the RHS as a CES overall productivity function with the sectoral efficiencies as inputs. This function has a CES equal to $1/(1 + \varphi)$, which turns negative for a large elasticity of substitution in production. Indeed, with $\varepsilon = 3$ and $\alpha = 1/3$, $1/(1+\varphi) = -3$. Therefore, the isoquants in the (A_c, A_d) — space are concave w.r.t. the origin. This means that one should either spur "clean" R&D or "dirty" R&D, but not both. Replacing A_{jt} with $A_{jt-1}(1+\gamma\eta s_{jt})$ turns Eq. (6) into an overall knowledge production function defined over the R&D inputs. Since the period resource constraint, $s_{ct} + s_{dt} = 1$, is symmetric, the choice of one corner solution over the other depends on the inherited relative efficiency, $(A_{dt-1}/A_{ct-1})^{-\varphi}$, and the tax term $(1+\tau_t)^{1-\varepsilon}$. A large inherited efficiency will favor the allocation of all scientists to the dirty energy sector. But as $\varepsilon > 1$, a sufficiently high tax rate will incite the reallocation of all scientists to the clean sector. The initial advantage of dirty energy production then explains why a steep hike in the tax rate is required to move the economy in *Regime* 1 from a "dirty" to a "clean" R&D focus. This also explains the "scissor"-shape in the upper-left panels of Figs. 2–4. Another implication of Eq. (6) is then that in the long-run, the productivity index B_t will grow at the same rate as A_c does, i.e., $\gamma\eta$.

In Regime 2, when there is full abatement, and therefore, $\tau_t \geq (A_{dt}/A_{at})^{1-\alpha}$, the overall productivity index can be rewritten as

$$B_t = (A_{ct}^{-\varphi} + [(A_{dt}^{-(1-\alpha)} + A_{at}^{-(1-\alpha)})^{-\frac{1}{1-\alpha}}]^{-\varphi})^{-\frac{1}{\varphi}}. \tag{7}$$

The RHS is now a two-level CES function over the three efficiency parameters A_{jt} $(j = c, d, a)$. At the lower level (square bracket term), the stocks of knowledge in the dirty energy and CCS sectors are aggregated into a productivity index for fully-abated fossil fuel energy. This function has an elasticity of substitution equal to $1/(2-\alpha)$, which amounts to $3/5$ when $\alpha = 1/3$. Therefore, efficiency in the production of dirty energy and abatement are strong complements, implying that R&D resources devoted to the fossil fuel sector should be allocated about evenly to the enhancement of both A_d and A_a. This explains the "$\succ$"-shape of the solid lines in the upper left panel of Fig. 4. The higher level CES function is defined on the efficiency for nonfossil fuel energy production and fully abated fossil energy generation. Like in the no-abatement case, this function has an elasticity of substitution equal to $1/(1+\varphi)$ which is negative for the selected coefficient values. It explains why we do not observe a scenario where researchers are shared between all three sectors. So far, we have provided a static explanation in terms of B, but the explanation in terms of growth rates of final consumption, which we have summarized in Appendix C, is very similar. In the long-run, the productivity index given by Eq. (7) will grow at a weighted average of the growth rates of A_d and A_a. Since researchers are shared evenly among these two sectors, B and, therefore, output and consumption grow only at the rate $\gamma\eta/2$. This also transpires from the lower right panel in Fig. 4: around year 180, both consumption paths cross, after which the dashed path (Regime 1) increases twice as fast as the solid one (Regime 2).

Figure 4 also reveals that Regime 1, compared to Regime 2, favors the environment in the medium run $(50 < t < 130)$ but performs worse in terms of consumption for $t \in [50, 180]$. The trade-off then becomes apparent. In Regime 1, the development and use of nonfossil fuel energy technology favors the environment in the medium run at the cost of a lower consumption level. In Regime 2, the gradual use and development of CSS technology allows for a higher consumption level in the medium run at the cost of higher temperature rise. For MC_a around 0.3, both strategies are equally good in terms of discounted utility. The fact that the clean strategy allows for a sustainable long-run growth rate that is twice as large is of minor importance in this respect: with an annual discount rate of 1.5%, the long-run ($t > 180$) is given very little weight.

We can sum up these findings as follows. First, the fact the clean and dirty energy are good substitutes induces a nonconvex input relationship between the productivities of the clean sector and dirty sector (possibly complemented with CCS) in the determination of the economy's overall productivity index. This means that the allocation of R&D resources is characterized by corner solutions and that changes in the nature of the solution only come about by large changes in policy and/or cost parameters. Second, the clean strategy allows for a long-run growth rate that is twice as large, but gives a lower consumption profile in the medium run, in favor of a smaller increase in temperature. This suggests two things. The first is that a lower discount rate will favor Regime 1 since this regime sustains a growth rate that is twice as high in the long run. This is also what we find: taking $\rho = 0.001$, Regime 1 dominates Regime 2 for MC_a as low as 0.1 (simulations not shown but available upon request). Second, Regime 1 will also be superior when more weight is given to the environment. This is conformed when we perform a sensitivity analysis w.r.t. the disaster temperature (or the maximal concentration of atmospheric CO_2 that needs to be avoided — see Section 7).

To the best of our knowledge, only Grimaud and Rouge (2014) draw attention to the adverse effects that CCS policies can have on the long-run growth rate. This is because CCS diverts labor away from research and lowers the output growth. Nevertheless, there is no directed technical change in their model.

7. Sensitivity Analysis

In this section, we perform three types of sensitivity analysis. First w.r.t. the elasticity of substitution, ε, next w.r.t. the disaster temperature rise, and finally w.r.t. the rate of technological progress.

7.1. *Elasticity of substitution between fossil fuel and nonfossil fuel energy*

The scenarios described earlier are based on an elasticity of substitution of 3, a benchmark number that was also used by van der Zwaan *et al.* (2002); Gerlagh and van der Zwaan (2003, 2004); Acemoglu *et al.* (2012). In a recent study, Papageorgiou *et al.* (2017) have estimated this crucial elasticity using sectoral data in a panel of 26 countries. Depending on the chosen specification, the elasticity is estimated very

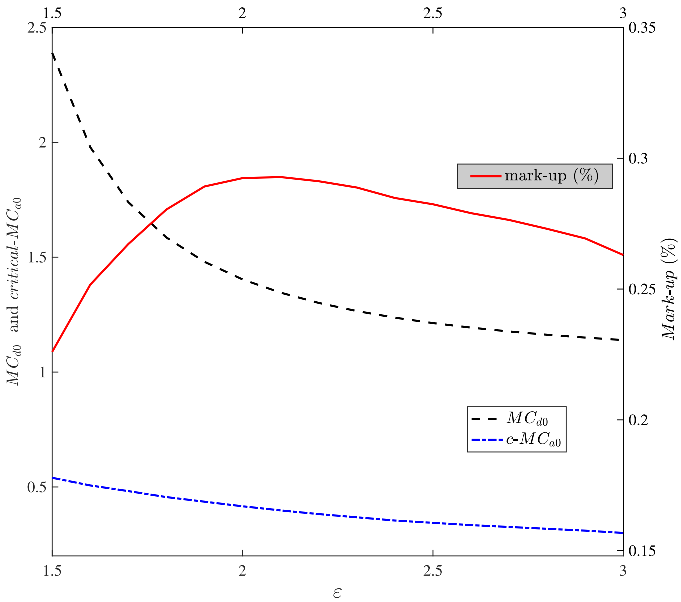


Figure 6. Critical values for MC_{a0} (i.e., c-MC_{a0}) and the mark-up $\frac{\mathrm{MC}_{a0}}{\mathrm{MC}_{d0}}$ below which Regime 1 is replaced by Regime 2

precisely between 1.7 and 2.8 for the electricity sector and 1.4 and 3.2 for the non-energy sectors. Hence, we perform a sensitivity analysis w.r.t. ε for the range [1.5, 3], and compute the critical values for MC_{a0} and the mark-up $\frac{\mathrm{MC}_{a0}}{\mathrm{MC}_{d0}}$ below which Regime 1 is replaced by Regime 2.[23] These values are given in Fig. 6. The red curve shows that the critical mark-up lies between 23% and 29%, and thus below the figures reported in row 3 of Table 2 above.

7.2. *Disaster temperature increase*

In all earlier scenarios, the increases in temperature around 2100 fall in the range of 4–5°C. On the other hand, the Paris Agreement aims at keeping global temperature rise well below 2°C and preferably to limit it to 1.5°C. We therefore inquiry about the optimal mitigation and R&D policy under stricter environmental constraints. We model these by reducing the disaster temperature parameter in the model, Δt_{dis} (maximal concentration of atmospheric CO_2), gradually from 6°C (1120 ppm) over 4°C (700 ppm) to 2°C (450 ppm).[24]

[23] Recall that the initial values for MC_{d0} and MC_{c0} depend on ε (see Appendix B).

[24] The parameter Δt_{dis} has two effects. The first is to shift the right-hand asymptote in Fig. 1 to the left. This is the intuitive effect: small increases in temperature will then result in larger damages. But there is a second effect: because the regeneration rate was calibrated as $\delta = \frac{1}{2}\frac{E_0}{S_{\mathrm{dis}}-S_0}$, a smaller Δt_{dis} translates (via a smaller S_{dis}) into a larger regeneration rate. This effect obviously comes from the forward-looking way of defining atmospheric quality in the ABHH-model and would be absent in backward-looking definitions (see Footnote 13). Therefore, in the sensitivity analysis w.r.t. Δt_{dis}, we keep δ at its original level of 0.0236.

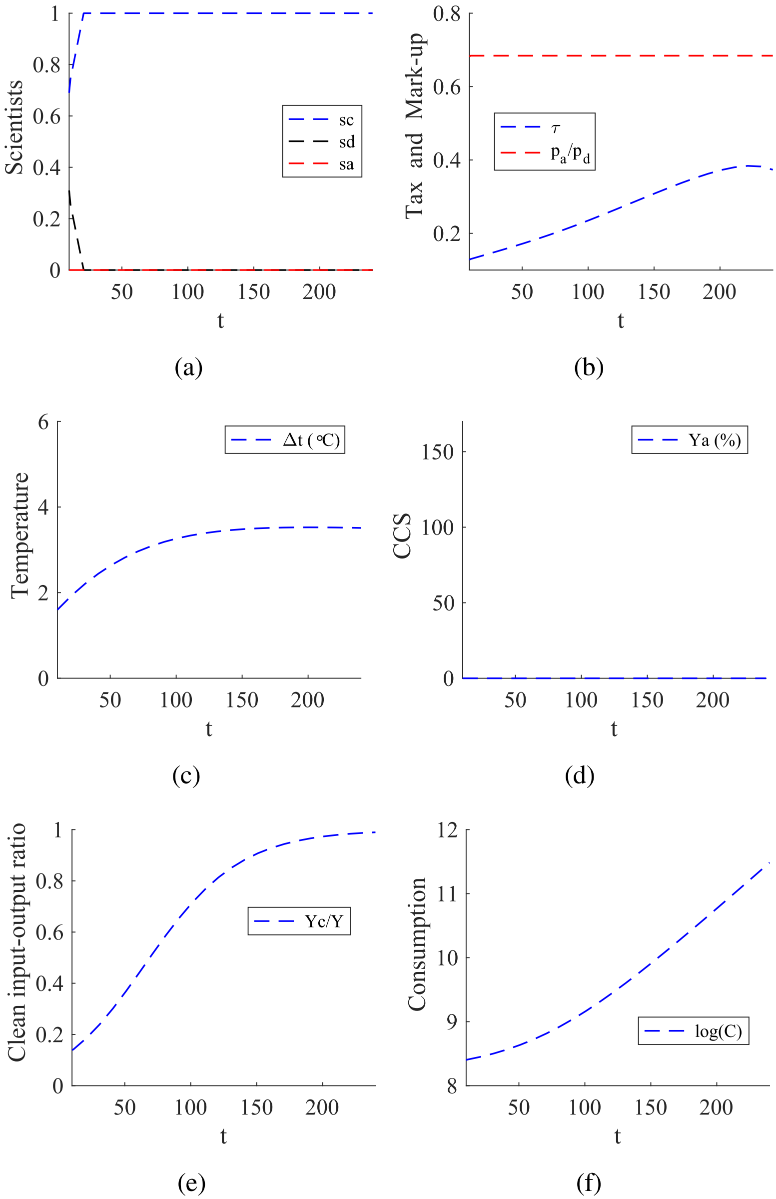


Figure 7. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = 0.684$ and $\Delta t_{\mathrm{dis}} = 4°\mathrm{C}$

We first consider the effects of a smaller scope for increases in temperature under the "realistic" benchmark for the marginal cost of CCS ($\mathrm{MC}_a = 0.684$). As shown in Figs. 7 and 8, and compared with Fig. 2, lower values for Δt_{dis} strongly advance the relocation of scientists from the dirty to the clean sector and at the same time raise the *ad valorem* tax rate even to such an extent under the $\Delta t_{\mathrm{dis}} = 2°\mathrm{C}$ scenario that CCS is activated immediately (Fig. 8). By 2050 (2100) less than 50% (20%) of energy is made

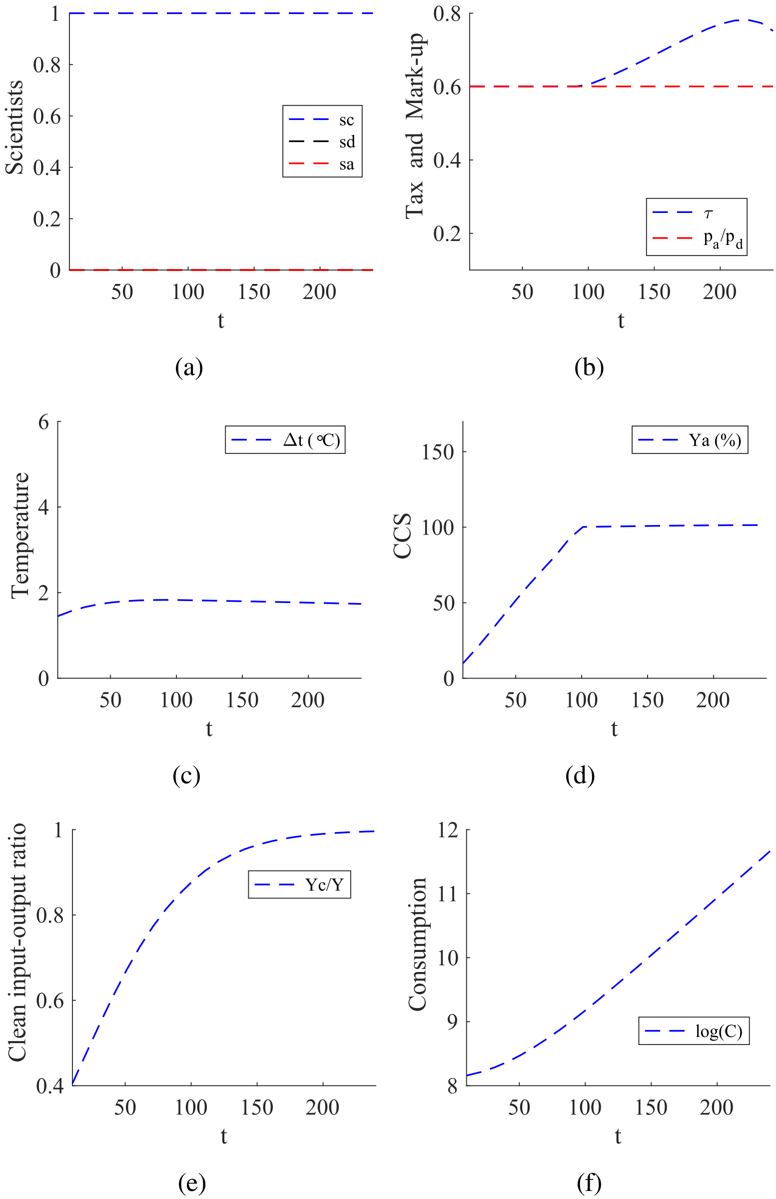


Figure 8. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = 0.684$, and $\Delta t_{\mathrm{dis}} = 2°\mathrm{C}$

up by fossil fuel energy. Furthermore, the emission cuts owing to the use of CCS drops from 25% in 2050 to 7% and 1% in 2100 and 2150, respectively (Fig. 9). Hence, CCS is only temporarily part of the solution.

In Fig. 10, we have combined the optimal pathways for the temperature change and for (log of) consumption. The effects of lower disaster temperature increases on the temperature pathway are intuitive. The consumption pathway rotates counterclockwise somewhere around 2090. Especially when restraining increases in temperature below

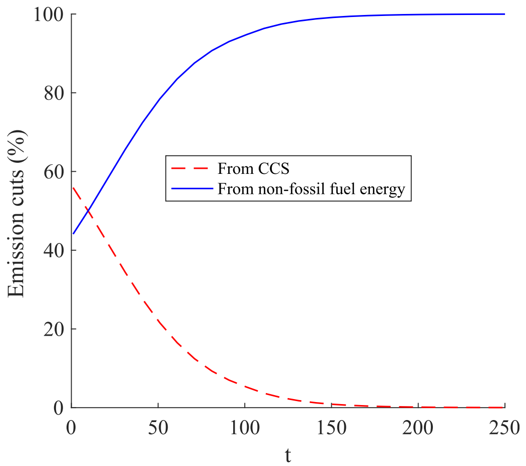
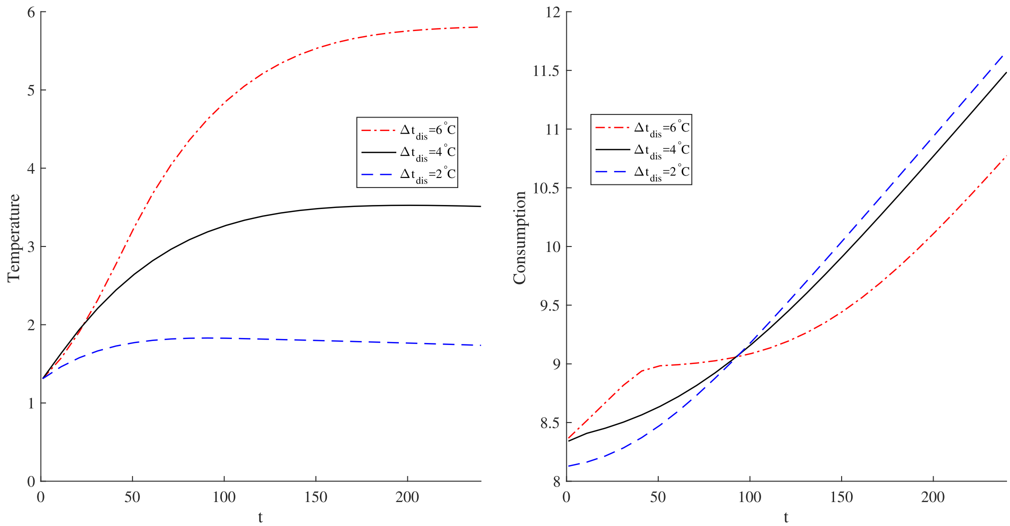


Figure 9. Emission cuts from CCS (Y_a) and nonfossil fuel energy (Y_c)

2°C, the figure shows that a big immediate consumption sacrifice should be made (of about 20%). In the long run, all three consumption pathways grow at 2% but the level is higher for the 2°C consumption pathway.

Earlier we identified a mark-up of CCS over the cost of fossil fuel energy of 26% ($\mathrm{MC}_a = 0.3$) as critical in the sense that at this marginal cost Regimes 1 and 2 are equivalent under $\Delta t_{\mathrm{dis}} = 6$ (cf. Fig. 4). As shown in Figs. 11 and 12, lower disaster temperatures makes Regime 1 superior to Regime 2 and the same pattern arises as in Figs. 7 and 8.



(a) (b)

Figure 10. Optimal pathways for temperature change and (log of) consumption

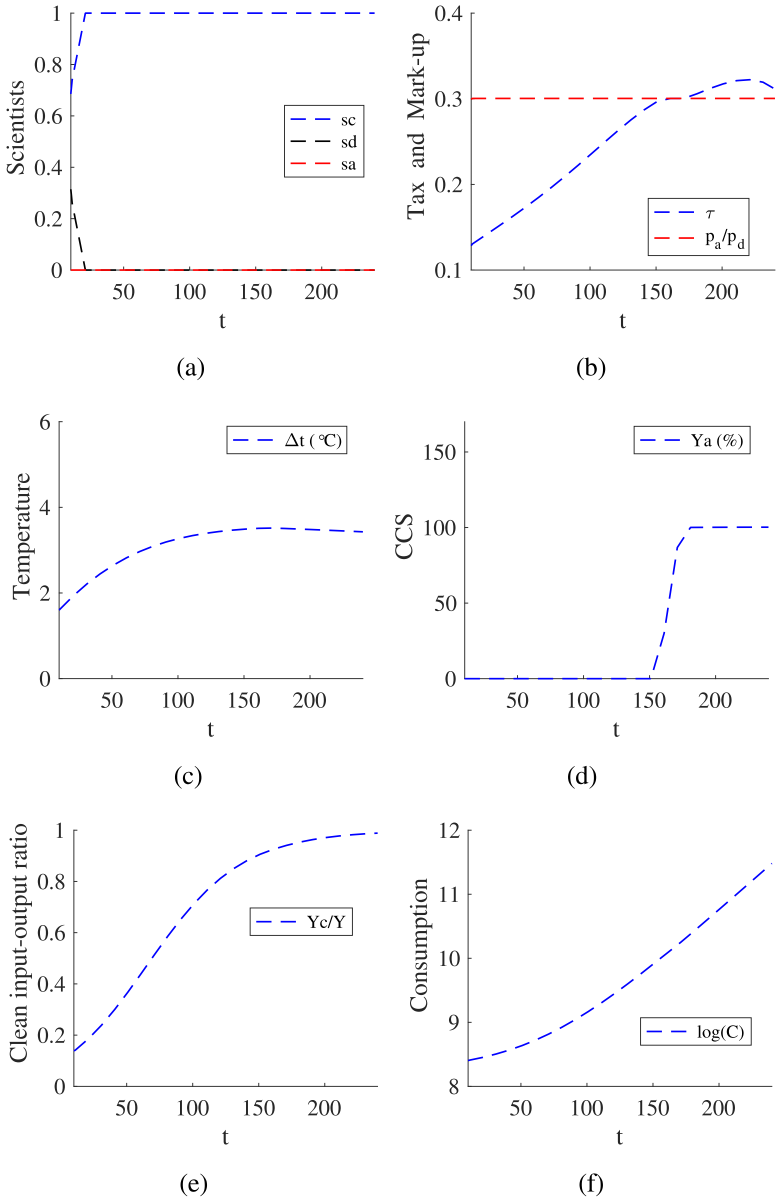


Figure 11. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = 0.3$, and $\Delta t_{\mathrm{dis}} = 4°\mathrm{C}$

7.3. *Success of innovation*

One may object to the assumption that all three sectors share the same rate of success in innovation. In comparison to mature technologies, technologies that are in their early stages of development may be expected to display higher rates of successful research. To test the implications of such a differentiation, we assume that the rate of success in innovation in the nonfossil fuel energy and CCS sectors exceeds temporarily

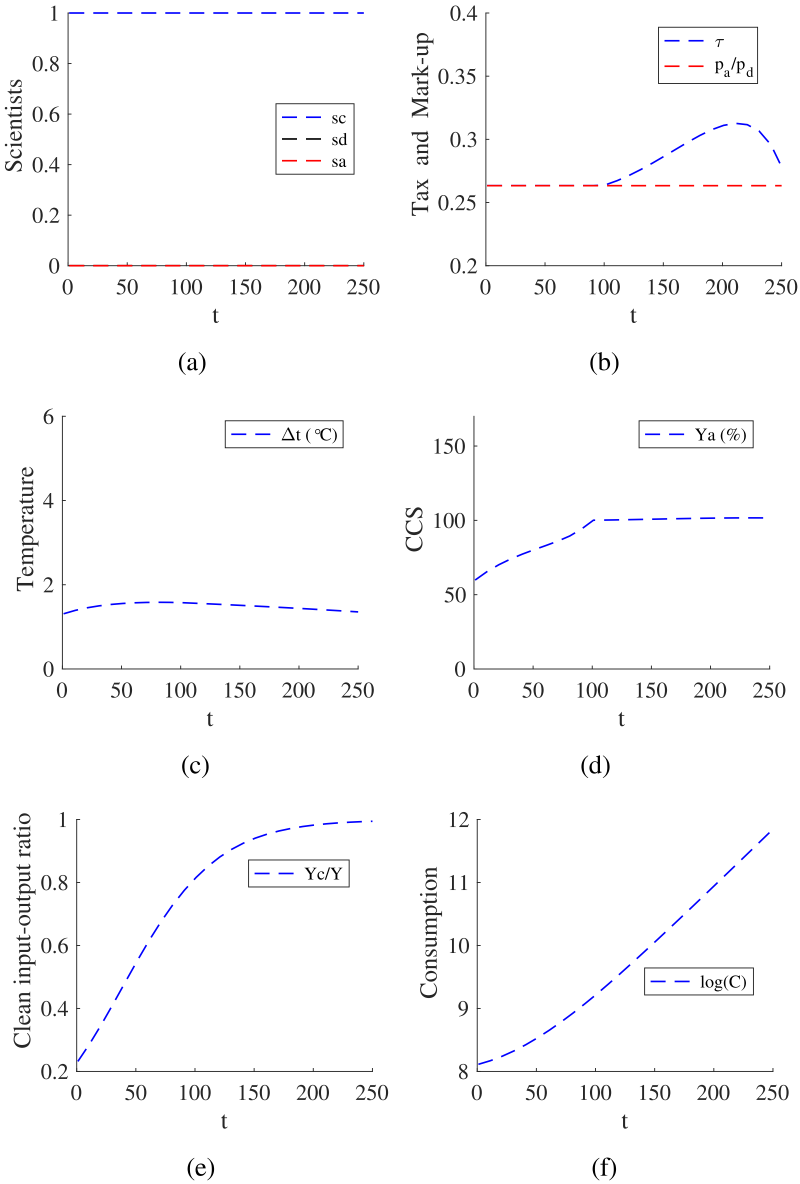


Figure 12. CD preferences, $\lambda = 0.1442$, $\mathrm{MC}_{a0} = 0.3$, $\Delta t_{\mathrm{dis}} = 2°\mathrm{C}$

(50 years) the rate in the fossil energy sector with 1% ($\eta_c = \eta_a = 0.03$ and $\eta_d = 0.02$ per annum). Figure 13 shows the results for the "critical" MC_a value of 0.30. Compared with Fig. 4, we see that Regime 2 is replaced by Regime 1: even though research on CCS technology is potentially more successful, the facts that CCS activity is complementary to the dirty energy production and the latter cannot grow at the same rate reduce the scope for CCS. As a consequence, it becomes optimal to fully allocate the researchers to the clean sector in the long run (despite a short reallocation after

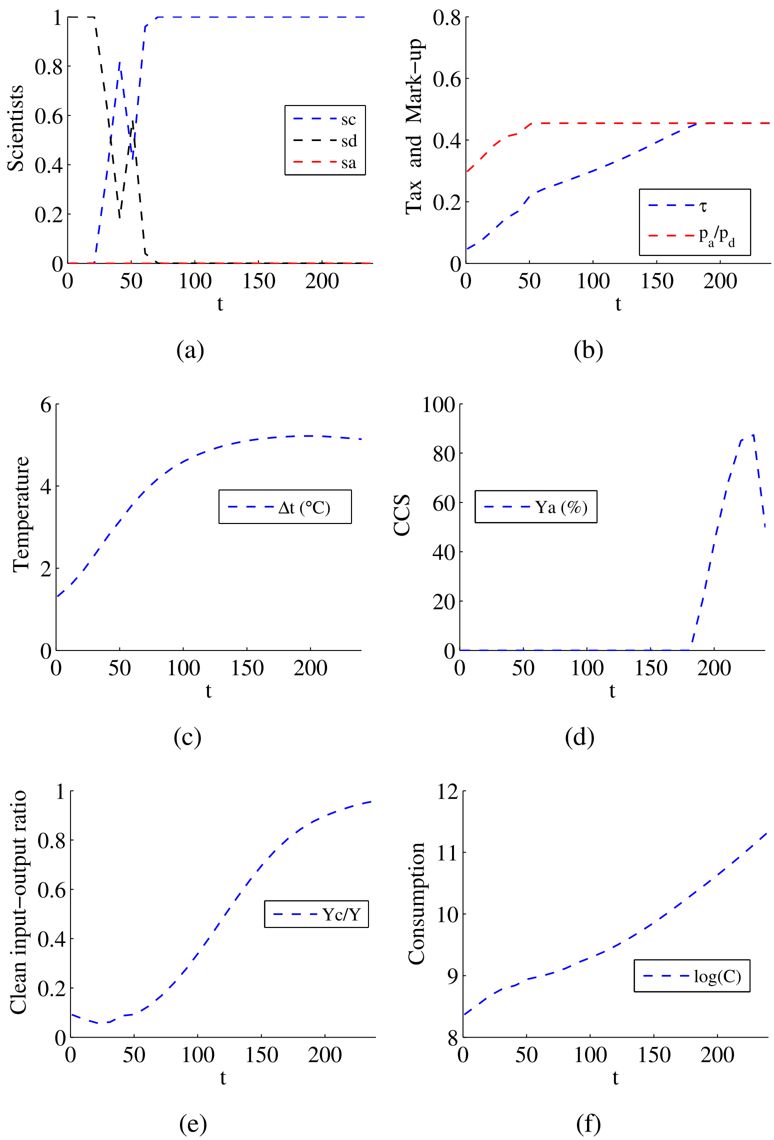


Figure 13. CD preferences, $\lambda = 0.1442$, $MC_{a0} = 0.3$, $\eta_c = \eta_a = 0.03$ for the first 50 years, followed by $\eta_c = \eta_a = 0.02$

50 years, when all three research sectors face the same potential rate of successful innovation again).

8. Conclusion

In recent decades, CCS has been considered as a promising strategy to curb CO_2 emissions and therefore to address the problem of global warming. Given the infancy of CCS technology, and the need for further RD&R, it is desirable to assess the optimality of this strategy not only on the basis of its current marginal cost, but also on the potential for improvements in cost efficiencies following R&D efforts in dirty energy, clean energy, and CCS sectors.

For this purpose, we made use of the directed technical change model of Acemoglu *et al.* (2012) by adding a sector responsible for CCS. Assuming that CCS competes for the same R&D resources as the fossil fuel and nonfossil fuel energy sectors, and that neither sector has any comparative advantage in transforming R&D into technological improvements, we have computed the Pareto-efficient time paths for production and research activity in each sector.

We identify a nonconvexity in the production of technological progress such that an optimal policy either devotes R&D resources to clean energy technology or to a joint development of dirty energy and CCS technology. Given the current estimates for the mark-up of the marginal cost of CCS over the marginal cost of fossil fuel energy, the latter strategy is dominated by the former. But this does not mean that CCS should never be employed. If the effect of CO_2 emissions on the environment are deemed sufficiently large, CCS should be deployed using today's technology, along with a gradual shift in focus from dirty energy to clean energy. Thus today's CCS technology should serve as a bridging technology while all R&D resources should be used to further develop the renewable energy technology.

Our analysis advises a current *ad valorem* tax on fossil fuel energy of 10% when considering weak environmental concerns (that is, when the aim is to keep global temperature rise below 6°C — see Fig. 2) and 60% when the environmental concerns are strong (that is, when the goal is to prevent global temperature from rising above 2°C — see Fig. 8). Globally, the current *ad valorem* tax is estimated at a mere 0.54%.[25] This low tax rate and the current high CCS mark-up cost can explain why the deployment of CCS has not been sufficiently high, despite a desire to keep temperature rise below 2°C.

The stylized model we worked with can be extended in several directions. One dimension is related to the dirty energy sector, which we assumed to be constrained by the amount of labor and capital devoted to transforming it into energy. Accordingly, we could have introduced a finite fossil fuel resource. However, as CCS depends on fossil fuel use, this will increase neither the scope for CCS nor the R&D devoted to this technology. Similarly, less favorable conditions for CCS (such as technically feasible capture rates below 100%, limited storage possibilities, and the risk of CO_2 leakage), while making the model more realistic, would only reduce the scope for this form of abatement activity and its technology.

[25] Carl and Fedor (2016) estimate the 2013 global carbon revenues to be 28.279 billion USD (see Tables 1 and 2 in Carl and Fedor, 2016). OECD/IEA (2015) reports that the world primary energy production for the dirty carriers in 2013 is 11022.37 millions of tonnes of oil equivalent. Using the price data for coal, crude oil, and natural gas provided by BP (2015), the 2013 world primary energy production from fossil fuels is calculated to yield 5269 billion US dollar, leading to an AVT of 0.0054.

Conversely, in our model renewable energy is being produced under rather optimistic circumstances, as we have assumed away any problems of intermittency and related problems regarding energy storage and transportation. Such issues impose additional costs to renewable energy production and suggest a lower degree of substitution between dirty and clean inputs. An interesting avenue for future research would therefore be to evaluate the scope for CCS when such favorable conditions are absent and ask whether research should be devoted to increase the effectiveness of renewables (e.g., allocating resources to improve the overall, or round-trip, efficiency of energy storage processes). In this model, emissions are only created by the use of fossil fuels in producing energy. But CO_2 is also emitted by carbon-intensive industries (iron, steel, cement, petrochemical processes). At the moment, CCS is considered as the only large-scale option to decarbonize such industries (IEA, 2013, p. 8).

One can also conjecture that if one primary energy type is or becomes dominant, the ease of substitution with alternative types would be reduced. This suggests an inverse-U-shaped relationship between the intensity of, say, fossil fuels, and the elasticity of substitution between fossil fuels and renewable energy (Gerlagh and Lise, 2005, p. 249).[26] As it will become more difficult to substitute away from dirty energy when this type is dominant, we expect that this would favor the role for CCS in our model. Another factor that can increase the role of CCS can be related to the lifetime of fossil fuel-burning infrastructure that can operate for up to 50 years (Kharecha and Hansen, 2013). A long operating lifetime can necessitate a much larger carbon price for replacement with nonfossil fuel energy technologies (Seto *et al.*, 2016) and spur the deployment of CCS technologies. The exploration of these issues is left for future research.

In our paper, we have confined ourselves to a search for the first-best policies. With a sufficiently broad set of instruments, these policies can be decentralized even in imperfect market economies. Accordingly, two market imperfections will need to be addressed: a tax on dirty production and subsidies to research activities because of R&D externalities.[27] In particular, targeting the right technologies to subsidize is a difficult task (Greaker *et al.*, 2018) and can lead to misdirection of resources. As this will lead to deviations from the optimal policy, a more aggressive tax policy can be called for to direct technical change and in turn avoid a climate disaster. A heavier use of carbon tax will lead to a higher demand for CCS and, indeed, can attract more researchers to improve the CCS technology. This will have interesting implications and will be crucial when linking the CCS technology better with actual policies.

Lastly, Hoel and Jensen (2012) show that if policy makers can at best commit to a future climate policy while failing to agree on adoption of a current policy, the reaction of fossil fuel owners (to advance the extraction of fossil fuels in time) may make it

[26]The advantage of Variable Elasticity of Substitution (VES) specification is that the substitution elasticity between the two energy sectors falls to 1 if one sector becomes dominant. Replacing CES by VES, however, comes at the cost of analytical complexity.

[27]In Acemoglu *et al.* (2012) machines are supplied by monopolistically competitive firms. In this case there is a third market imperfection. Therefore, machine users will need to be subsidized.

more desirable to aim at a faster technical progress in abatement rather than in renewable energy production. The consequences of such restrictions for our model would be worthwhile investigating.

Acknowledgments

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Appendix A. Solution of the Model

The Lagrangian function for the planning problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t W_t,$$

where

$$\begin{aligned} W_t = {} & U(Y_t - \psi \sum_{j=a,c,d} x_{jt} \tilde{F}(S_t)) + \pi_t \left[\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} - Y_t \right] \\ & + \sum_{j=a,c,d} \pi_{jt} \left[L_{jt}^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha} - Y_{jt} \right] + w_t \left[1 - \sum_{j=a,c,d} L_{jt} \right] \\ & + v_t \left[1 - \sum_{j=a,c,d} s_{jt} \right] + \sum_{j=a,c,d} \mu_{jt} \left[\left(1 + \gamma \eta_j s_{jt} \right) A_{jt-1} - A_{jt} \right] \\ & + \omega_t [S_t - S_{t-1} - \xi (Y_{dt-1} - Y_{at-1}) + \delta (S_{\text{dis}} - S_{t-1})] + \phi_t [Y_{dt} - Y_{at}]. \end{aligned}$$

Thus W_t is the undiscounted period t welfare, π_t is the social value of final production in period t, $w_t(v_t)$ is the shadow value of labor (research) in period t, μ_{jt} is the social value of productivity in sector j in period t, and ω_t is the social value of the environment in period t. When writing the quality index of the environment as a function of the stock of CO_2, we have subsumed the relationship through the increase in temperature, Δt.

The FOC w.r.t. Y_t shows that $\pi_t = U_{ct}$. The FOC w.r.t. S_t yields $U_{F_t}(-\tilde{F}'_t) = \omega_t - (1+\delta)\beta\omega_{t+1}$. This is a forward-looking equation that can be solved for ω_t, the social value of a one unit improvement of the environment in t, as

$$\omega_t = \sum_{k=0}^{\infty} \beta^k (1+\delta)^k U_{F_k}(-\tilde{F}'_k).$$

Improving the environment today thus generates a stream of future benefits (since $-\tilde{F}'_k > 0$).

We now solve for the remaining decision variables. First, note that because of the CES specification, both the clean and dirty inputs will be used in strictly positive quantities. For the clean input, we obtain the optimality condition

$$\mathrm{MP}_{ct} = \widehat{p}_{ct} \stackrel{\text{def}}{=} \frac{\pi_{ct}}{\pi_t},$$

i.e., the equality of its marginal product, MP_{ct}, with its social price. A similar condition holds for the dirty input, corrected for the environmental externality

$$\mathrm{MP}_{dt} = \widehat{p}_{dt} + \xi\beta\frac{\omega_{t+1}}{\pi_t} - \frac{\phi_t}{\pi_t}, \tag{A.1}$$

where $\widehat{p}_{dt} \stackrel{\text{def}}{=} \frac{\pi_{dt}}{\pi_t}$ is the social price of the dirty input. The term $\xi\beta\frac{\omega_{t+1}}{\pi_t}$ is equivalent to a tax on the use of the dirty input in a decentralized solution. It ensures a more moderate use of the dirty input than the equality of MP_{dt} with $\widehat{p}_{dt}$ would call for. The extra term $\frac{\phi_t}{\pi_t}$ is due to abatement. Before interpreting it, we give the FOC w.r.t. Y_{at}

$$-\pi_{at} + \xi\beta\omega_{t+1} - \phi_t \le 0,$$

with equality when $Y_{at} > 0$. Dividing through by π_t and defining $\widehat{p}_{at} \stackrel{\text{def}}{=} \frac{\pi_{at}}{\pi_t}$, we can write this as

$$\widehat{p}_{at} \ge \xi\beta\frac{\omega_{t+1}}{\pi_t} - \frac{\phi_t}{\pi_t}.$$

If any abatement is suboptimal, $Y_{at} = 0 < Y_{dt}$, and $\widehat{p}_{at} \ge \xi\beta\frac{\omega_{t+1}}{\pi_t}$; the social marginal cost of abatement is too high compared with its social marginal benefit. However, suppose that abatement is optimal, then either there is partial abatement, $0 < Y_{at} \le Y_{dt}$, in which case $\widehat{p}_{at} = \xi\beta\frac{\omega_{t+1}}{\pi_t}$, or there is full abatement, $Y_{at} = Y_{dt}$, in which case $\widehat{p}_{at} \le \xi\beta\frac{\omega_{t+1}}{\pi_t}$. In this last case, the social marginal benefit is larger than the social marginal cost, but the welfare programme is constrained by the fact that abatement can only apply to contemporaneous emissions, not to CO_2 emitted in previous periods (i.e., it is not possible to remove previously emitted CO_2 from the atmosphere). If this is the case, then social welfare may be increased by expanding dirty input production beyond the level where $\mathrm{MP}_{dt} = \widehat{p}_{dt} + \xi\beta\frac{\omega_{t+1}}{\pi_t}$. Indeed, then:

$$\mathrm{MP}_{dt} = \widehat{p}_{dt} + \widehat{p}_{at}. \tag{A.2}$$

CO_2 abatement is merely an additional social cost. Thus, we can conclude that

$$\mathrm{MP}_{dt} = \widehat{p}_{dt} + \min\left\{\hat{p}_{at}, \xi\beta\frac{\omega_{t+1}}{\pi_t}\right\}, \tag{A.3}$$

$$= \widehat{p}_{dt} + \min\{\hat{p}_{at}, \tau_t\hat{p}_{dt}\}. \tag{A.4}$$

Having determined the optimality conditions for Y_{jt}, we now consider the use of labor and physical capital. Both inputs are required in positive amounts. For labor, the value of the marginal product of labor in the production of sector j must equal the social wage rate $\widehat{w}_t \stackrel{\text{def}}{=} \frac{w_t}{\pi_t}$

$$\widehat{p}_{jt}\mathrm{MP}_{Ljt} = \widehat{w}_t, \text{or} \tag{A.5}$$

$$(1-\alpha)\widehat{p}_{jt}A_{jt}^{1-\alpha}\left(\frac{x_{jt}}{L_{jt}}\right)^{\alpha} = \widehat{w}_t. \tag{A.6}$$

Likewise, for machines

$$\widehat{p}_{jt}\mathrm{MP}_{xjt} = \psi, \text{or} \tag{A.7}$$

$$\alpha\widehat{p}_{jt}A_{jt}^{1-\alpha}\left(\frac{L_{jt}}{x_{jt}}\right)^{1-\alpha} = \psi, \tag{A.8}$$

where ψ is the (exogenously given) amount of final goods necessary to build one machine.

Finally, we determine the allocation of scientists, and the production of knowledge. The FOC w.r.t. s_{jt} is

$$\frac{\mu_{jt}}{\pi_t}\gamma\eta_j A_{jt-1} \le \widehat{v}_t \stackrel{\text{def}}{=} \frac{v_t}{\pi_t},$$

with equality whenever $s_{jt} > 0$. The left-hand side (LHS) is the social price of sector j knowledge, $\frac{\mu_{jt}}{\pi_t}$, times the marginal knowledge production of an additional researcher. The RHS is the social wage rate of a researcher.

The final set of FOCs characterizes the allocation of productivity improvements in the different sectors across time. The FOC w.r.t. A_{jt} reads

$$\widehat{p}_{jt}(1-\alpha)L_{jt}^{1-\alpha}\left(\frac{x_{jt}}{A_{jt}}\right)^{\alpha} = \frac{\mu_{jt}}{\pi_t} - \beta\frac{\mu_{jt+1}}{\pi_{t+1}}\frac{\pi_{t+1}}{\pi_t}(1+\gamma\eta_j s_{jt+1}).$$

The LHS is the value of the marginal product of newly acquired knowledge on the use of machines. Using (A.8), an optimal allocation of knowledge implies that the social price of sector j knowledge, $\frac{\mu_{jt}}{\pi_t}$, must evolve according to the rule

$$(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}L_{jt}\widehat{p}_{jt}^{\frac{1}{1-\alpha}} = \frac{\mu_{jt}}{\pi_t} - \beta\frac{\mu_{jt+1}}{\pi_{t+1}}\frac{\pi_{t+1}}{\pi_t}(1+\gamma\eta_j s_{jt+1}).$$

Multiplying through by $\pi_t A_{jt}$ and making use of $A_{jt+1} = (1 + \gamma\eta_j s_{jt+1})A_{jt}$ gives

$$\mu_{jt}A_{jt} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}\pi_t \widehat{p}_{jt}^{\frac{1}{1-\alpha}} A_{jt} + \beta\mu_{jt+1}A_{jt+1}.$$

The social value of acquired knowledge in sector j at time t is the value of A_{jt} priced at its marginal product plus the "standing on the shoulder of giants" effect (future knowledge builds on today's knowledge). Using the forward operator F, multiplying through by $\gamma\eta_j \frac{A_{jt-1}}{A_{jt}}$ and making use of $A_{jt} = (1 + \gamma\eta_j s_{jt})A_{jt-1}$ result in

$$\begin{aligned}\frac{\mu_{jt}}{\pi_t}\gamma\eta_j A_{jt-1} &= \frac{1}{\pi_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt}}\frac{1}{1-\beta F}(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}\pi_t \widehat{p}_{jt}^{\frac{1}{1-\alpha}} A_{jt},\\ &= \frac{1}{\pi_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt}}(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \sum_{k=0}^{\infty}\beta^k L_{jt+k}\pi_{t+k}\widehat{p}_{jt+k}^{\frac{1}{1-\alpha}} A_{jt+k},\end{aligned} \tag{A.9}$$

so that the social value of allocating an extra researcher to sector j is given by the discounted sum of future knowledge levels, appropriately valued and weighted.

Solving (A.8) for x_{jt} gives

$$x_{jt} = \left(\alpha\frac{\widehat{p}_{jt}}{\psi}\right)^{\frac{1}{1-\alpha}} A_{jt}L_{jt}, \tag{A.10}$$

which can be plugged into (A.6) to yield the social price of sector j output, as a weighted average of the exogenous machine price, ψ, and the shadow price of labor, $\widehat{w}_{jt}$

$$\widehat{p}_{jt} = \frac{1}{\mathcal{A}}\frac{1}{A_{jt}^{1-\alpha}}\widehat{w}_t^{1-\alpha}\psi^{\alpha}, \tag{A.11}$$

where $\mathcal{A} \stackrel{\text{def}}{=} \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Hence, at an optimum, $\widehat{p}_{jt}$ will equal the social marginal cost of sector j output.

Next, the FOCs for Y_{ct} and Y_{dt} can be used to relate these input levels to aggregate output, Y_t and the shadow prices of the inputs

$$Y_{ct} = Y_t\widehat{p}_{ct}^{-\varepsilon} \quad \text{and}$$

$$Y_{dt} = Y_t[\widehat{p}_{dt} + \min\{\hat{p}_{at}, \tau_t\hat{p}_{dt}\}]^{-\varepsilon}$$

$$= Y_t\widehat{p}_{dt}^{-\varepsilon}\left[1 + \min\left\{\frac{\hat{p}_{at}}{\hat{p}_{dt}}, \tau_t\right\}\right]^{-\varepsilon}, \tag{A.12}$$

$$= Y_t\widehat{p}_{dt}^{-\varepsilon}\left[1 + \min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{-\varepsilon}, \tag{A.13}$$

where the last equality follows from (A.11). Making use of the final good production function, we obtain

$$1 = \widehat{p}_{ct}^{1-\varepsilon} + \widehat{p}_{dt}^{1-\varepsilon}\left[1 + \min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{1-\varepsilon}.$$

As $\widehat{p}_{jt}$ $(j = c, d, a)$ are given by (A.11)

$$1 = \frac{1}{\mathcal{A}^{1-\varepsilon}}\widehat{w}_t^{\varphi}\psi^{\alpha(1-\varepsilon)}\left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1 + \min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{1-\varepsilon}\right),$$

where $\varphi \overset{\text{def}}{=} (1-\alpha)(1-\varepsilon)$.

Hence, we can solve for the social value of the wage rate

$$\begin{aligned}\widehat{w}_t &= \mathcal{A}^{\frac{1}{1-\alpha}}\psi^{-\frac{\alpha}{1-\alpha}}\left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1 + \min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{1-\varepsilon}\right)^{-\frac{1}{\varphi}}, \\ &= \mathcal{A}^{\frac{1}{1-\alpha}}\psi^{-\frac{\alpha}{1-\alpha}}B_t,\end{aligned} \tag{A.14}$$

thereby implicitly defining the "sector average" productivity parameter B_t as

$$B_t \overset{\text{def}}{=} \left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1 + \min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{1-\varepsilon}\right)^{-\frac{1}{\varphi}}. \tag{A.15}$$

From (A.14) and (A.11), the social prices of the two inputs as well as the price of abatement are then

$$\widehat{p}_{jt} = \left(\frac{B_t}{A_{jt}}\right)^{1-\alpha} \quad (j = c, d, a). \tag{A.16}$$

Machine use in sector j can be obtained from (A.10) and (A.16)

$$x_{jt} = \widehat{p}_{jt}^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}}A_{jt}L_{jt} = \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}}B_t L_{jt}, \tag{A.17}$$

and therefore the aggregate machine cost (the share of final good production used for capital) is

$$\mathrm{AMC}_t \overset{\text{def}}{=} \sum_{j=c,d,a}\psi x_{jt} = \psi\left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}}B_t\sum_{j=c,d,a}L_{jt} = \psi^{-\frac{\alpha}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}B_t, \tag{A.18}$$

where the last equality follows from the normalization of the labor supply to one.

To find the levels of production in the three sectors, we plug the solution for x_{jt} (A.17) into the production function, yielding

$$Y_{jt} = A_{jt}L_{jt}\left(\alpha\frac{\widehat{p}_{jt}}{\psi}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}L_{jt}A_{jt}^{1-\alpha}B_t^{\alpha} \quad (j = c, d, a).$$

Therefore $L_{jt} = (\frac{\alpha}{\psi})^{-\frac{\alpha}{1-\alpha}} A_{jt}^{\alpha-1} B_t^{-\alpha} Y_{jt}$, which allows us to write (A.9) as

$$\begin{aligned}\frac{\mu_{jt}}{\pi_t}\gamma\eta_j A_{jt-1} &= \frac{1}{\pi_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt}}(1-\alpha)\sum_{k=0}^{\infty}\beta^k A_{jt+k}^{\alpha} B_{t+k}^{-\alpha} Y_{jt+k}\pi_{t+k}\hat{p}_{jt+k}^{\frac{1}{1-\alpha}},\\ &= \frac{1}{\pi_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt}}(1-\alpha)\sum_{k=0}^{\infty}\beta^k \pi_{t+k}\hat{p}_{jt+k} Y_{jt+k},\end{aligned}$$

where the second equality follows from (A.16). This is expression (5) in the text.

On the other hand, (A.12) and (A.13) together with (A.16) give

$$Y_{ct} = Y_t\left(\frac{B_t}{A_{ct}}\right)^{-\varepsilon(1-\alpha)} \quad \text{and} \tag{A.19}$$

$$Y_{dt} = Y_t\left(\frac{B_t}{A_{dt}}\right)^{-\varepsilon(1-\alpha)}\left[1+\min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{-\varepsilon}. \tag{A.20}$$

The last three expressions now allow us to write the labor balance equation as

$$\begin{aligned}&A_{ct}^{-\varphi} B_t^{-(1-\varphi)} Y_t + A_{dt}^{-\varphi} B_t^{-(1-\varphi)}\left[1+\min\left\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t\right\}\right]^{-\varepsilon} Y_t\\ &\quad + Y_{at} A_{at}^{-(1-\alpha)} B_t^{-\alpha} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}.\end{aligned} \tag{A.21}$$

We now look at the three possibilities. The first is where there is full abatement, $Y_{at} = Y_{dt}$, such that $\min\{(\frac{A_{dt}}{A_{at}})^{1-\alpha}, \tau_t\} = (\frac{A_{dt}}{A_{at}})^{1-\alpha}$. In that case

$$Y_{at} = Y_{dt} = Y_t\left(\frac{B_t}{A_{dt}}\right)^{-\varepsilon(1-\alpha)}\left[1+\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right]^{-\varepsilon}.$$

Making use of these values for Y_{at} and Y_{dt} in the labor balance equation (A.21) reduces the latter to

$$\left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} B_t^{-(1-\varphi)} B_t^{-\varphi} Y_t = 1,$$

so that

$$\begin{aligned}Y_t^{FA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} B_t,\\ Y_{ct}^{FA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} B_t^{\varphi+\alpha} A_{ct}^{1-(\varphi+\alpha)},\\ Y_{dt}^{FA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} B_t^{\varphi+\alpha} A_{dt}^{1-(\varphi+\alpha)}\left[1+\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right]^{-\varepsilon}, \quad \text{and}\\ Y_{at}^{FA} &= Y_{dt}^{FA}.\end{aligned} \tag{A.22}$$

In the second case, there is partial abatement such that $0 < Y_{at} < Y_{dt}$ and $\min\{(\frac{A_{dt}}{A_{at}})^{1-\alpha}, \tau_t\} = (\frac{A_{dt}}{A_{at}})^{1-\alpha} = \tau_t$. Now (A.21) becomes

$$\{A_{ct}^{-\varphi}B_t^{-(1-\varphi)}Y_t + A_{dt}^{-\varphi}B_t^{-(1-\varphi)}[1+\tau_t]^{-\varepsilon}Y_t + Y_{at}A_{at}^{-(1-\alpha)}B_t^{-\alpha}\} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}},$$

yielding

$$Y_t^{\mathrm{PA}} = \left\{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} - Y_{at}A_{at}^{-(1-\alpha)}B_t^{-\alpha}\right\}\frac{B_t^{1-\varphi}}{[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}[1+\tau_t]^{-\varepsilon}]}, \tag{A.23}$$

$$Y_{ct}^{\mathrm{PA}} = Y_t^{\mathrm{PA}}\left(\frac{B_t}{A_{ct}}\right)^{-\varepsilon(1-\alpha)}, \tag{A.24}$$

$$Y_{dt}^{\mathrm{PA}} = Y_t^{\mathrm{PA}}\left(\frac{B_t}{A_{dt}}\right)^{-\varepsilon(1-\alpha)}[1+\tau_t]^{-\varepsilon}. \tag{A.25}$$

For this to be compatible with partial abatement, we need $Y_{at} \leq Y_{dt}^{\mathrm{PA}}$, which can be shown to be equivalent with

$$Y_{at} \leq \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\frac{A_{dt}^{\varepsilon(1-\alpha)}B_t^{\alpha}[1+\tau_t]^{-\varepsilon}}{\{A_{ct}^{-\varphi} + A_{dt}^{-\varphi}[1+\tau_t]^{1-\varepsilon}\}}. \tag{A.26}$$

Given partial abatement is optimal, we have $(\frac{A_{dt}}{A_{at}})^{1-\alpha} = \tau_t$, and this condition reduces to

$$Y_{at} \leq \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\frac{A_{dt}^{\varepsilon(1-\alpha)}B_t^{1-\varepsilon(1-\alpha)}}{[1+\tau_t]^{\varepsilon}}. \tag{A.27}$$

In the third case, there is no abatement: $Y_{at} = 0$ and $\min\{(\frac{A_{dt}}{A_{at}})^{1-\alpha}, \tau_t\} = \tau_t$. The equilibrium value for Y_t is then found by setting $Y_{at} = 0$ in (A.23)

$$Y_t^{\mathrm{NA}} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\frac{B_t^{1-\varphi}}{[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}(1+\tau_t)^{-\varepsilon}]}, \tag{A.28}$$

and

$$\begin{aligned}
Y_{ct}^{\mathrm{NA}} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\frac{B_t^{\alpha}A_{ct}^{\varepsilon(1-\alpha)}}{[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}(1+\tau_t)^{-\varepsilon}]},\\
Y_{dt}^{\mathrm{NA}} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\frac{B_t^{\alpha}A_{dt}^{\varepsilon(1-\alpha)}}{[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}(1+\tau_t)^{-\varepsilon}]}(1+\tau_t)^{-\varepsilon}, \quad \text{and}\\
Y_{at}^{\mathrm{NA}} &= 0.
\end{aligned}$$

When solving the model, we search for a sequence $\{Y_{at}, \tau_t, s_{ct}, s_{dt}\}_{t=1}^{T}$ (where T is large) that maximizes

$$\sum_{t=1}^{T} \beta^t U(Y_t^{\mathrm{PA}}(Y_{at}) - \mathrm{AMC}_t, \tilde{F}(S_{t-1} + \xi(Y_{dt-1} - Y_{at-1}) - \delta(S_{\mathrm{dis}} - S_{t-1}))),$$

subject to the equality constraints (A.18), (A.24) and (A.25), $s_{at} = 1 - s_{ct} - s_{dt}$, $A_{jt} = (1 + \gamma\eta_j s_{jt})A_{jt-1}$ (all t and j), the nonlinear inequality constraint (A.26), the non-negativity constraint $Y_t^{\mathrm{PA}}(Y_{at}) - \mathrm{AMC}_t \geq 0$, and with the initial productivity levels A_{j0} given.

Appendix B. Calibration of the Model

Without any policy intervention in the base period, the laissez-faire levels for clean and dirty input production are

$$Y_{c0} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_0^{\alpha} A_{c0}^{\varepsilon(1-\alpha)}}{A_{c0}^{-\varphi} + A_{d0}^{-\varphi}} \quad \text{and} \quad Y_{dt} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_0^{\alpha} A_{d0}^{\varepsilon(1-\alpha)}}{A_{c0}^{-\varphi} + A_{d0}^{-\varphi}}.$$

This system can be solved for A_{d0} and A_{c0}

$$A_{d0} = \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{d0} \left[1 + \left(\frac{Y_{d0}}{Y_{c0}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{\frac{\alpha+\varphi}{\varphi}},$$
$$A_{c0} = \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{c0} \left[1 + \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{\frac{\alpha+\varphi}{\varphi}}.$$

As in AABH, we have used the values for world primary energy production by energy type during the period 2002–2006 (EIA, 2008, Table 11.1) and doubled them. Dirty energy (coal, natural gas, crude oil, and natural gas plant liquids) yield 3786 QBTU, while clean energy (solar and wind power, nuclear electric power, hydro-electric power, geothermal, electricity generation from wood and waste) provide 615 QBTU.[28] Under the assumptions that $\alpha = \frac{1}{3}, \varepsilon = 3$ (and therefore $\varphi = -\frac{4}{3}$), $\rho = 0.015$, and the normalization $\psi = \alpha^2$ (Acemoglu *et al.*, 2012), we obtain the following estimates for A_{d0}, A_{c0}, and B_0: $A_{d0} = 2658$, $A_{c0} = 1072$, and $B_0 = 3232$.

Appendix C. Long-Run Growth Scenarios

In this section we inquire about the optimal R&D policy in the steady state. We distinguish between a steady state where there is no abatement and one where there is

[28]The corresponding values for 2002–2006, adopted by AABH were 1893.25 and 307.77, respectively. We double the world primary energy production figures to obtain the same reference sectoral marginal costs of production as in AABH (cf. MC_{d0} and MC_{c0} on p. 13 in Section 4).

full abatement. We omit the time index and denote the growth rate in variable y as $\dot{y}$, i.e., $\dot{y} = \frac{\mathrm{d}\log y}{\mathrm{d}t}$.

When there is no abatement, $\tau < (\frac{A_d}{A_a})^{1-\alpha}$ and the productivity index (A.15) reduces to

$$B(\ell) \overset{\text{def}}{=} (A_c^{-\varphi} + A_d^{-\varphi}[1+\tau]^{\ell-\varepsilon})^{-\frac{1}{\varphi}} \quad \text{with } \ell = 1.$$

Then from (A.28), it follows that:

$$Y^{\text{NA}} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B(1)^{1-\varphi}}{B(0)^{1-\varphi}}.$$

Since $\text{AMC} = (\frac{\alpha}{\psi})^{\frac{\alpha}{1-\alpha}} B(1)$ and since $C = Y - \text{AMC}$, the growth rate in consumption is given by

$$\begin{aligned}
\dot{C} &= \frac{Y}{C}\dot{Y} - \frac{\text{AMC}}{C}\dot{\text{AMC}},\\
&= \left[\kappa_c(1) - \frac{Y}{C}\varphi(\kappa_c(1) - \kappa_c(0))\right]\dot{A}_c + \left[\kappa_d(1) - \frac{Y}{C}\varphi(\kappa_d(1) - \kappa_d(0))\right]\dot{A}_d\\
&\quad - \left[\left(\frac{1}{1-\alpha} - \frac{Y}{C}\right)\kappa_d(1) + \frac{Y}{C}\varepsilon(\kappa_d(1) - \kappa_d(0))\right]\frac{\tau}{1-\tau}\dot{\tau},
\end{aligned}$$

where $\kappa_c(\ell) \overset{\text{def}}{=} \frac{A_c^{-\varphi}}{A_c^{-\varphi} + A_d^{-\varphi}(1+\tau)^{\ell-\varepsilon}}$ and $\kappa_d(\ell) \overset{\text{def}}{=} 1 - \kappa_c(\ell)$.

Tedious manipulations then show that $\frac{C}{Y} - (1-\alpha) = \alpha\kappa_d(1)\frac{\tau}{1+\tau}$. And since $1 - \frac{\kappa_d(0)}{\kappa_d(1)} = \frac{\tau}{1+\tau}\kappa_c(0)$, the growth rate in C can be written as

$$\begin{aligned}
\dot{C} &= \left[\kappa_c(1) - \frac{Y}{C}\varphi(\kappa_c(1) - \kappa_c(0))\right]\dot{A}_c + \left[\kappa_d(1) - \frac{Y}{C}\varphi(\kappa_d(1) - \kappa_d(0))\right]\dot{A}_d\\
&\quad - \frac{Y}{C}\kappa_d(1)\left[\frac{\alpha}{1-\alpha}\kappa_d(1) + \varepsilon\kappa_c(0)\right]\frac{\tau}{1+\tau}\dot{\tau}.
\end{aligned}$$

Logarithmic differentiation of $\kappa_c(\ell)$ yields

$$\dot{\kappa}_c(\ell) = (-\varphi)\kappa_d(\ell)\left[\dot{A}_c - \dot{A}_d - \frac{\tau}{1-\tau}\dot{\tau}\right].$$

Thus when all researchers are allocated to the clean energy sector ($s_c = 1, s_d = 0$), $\dot{A}_c = \gamma\eta$ and $\dot{A}_d = 0$. Then $\kappa_c(\ell) \to 1$ and $\kappa_d(\ell) \to 0$. Since $\kappa_d(1) \to 0$, the term with $\dot{\tau}$ in the expression for $\dot{C}$ will vanish. Hence $\dot{C} \to \dot{A}_c = \gamma\eta$. On the other hand, if all researchers would be allocated to the dirty energy sector, $\dot{A}_d = \gamma\eta$ and $\dot{A}_c = 0$. Then $\kappa_d(\ell) \to 1$ while $\kappa_c(\ell) \to 0$, and the long-term growth rate in consumption is given by

$$\dot{C} = \gamma\eta - \frac{Y}{C}\frac{\alpha}{1-\alpha}\frac{\tau}{1+\tau}\dot{\tau}.$$

Since the tax rate has to grow to contain the environmental consequences of dirty energy production, the growth rate is less than in the clean R&D scenario. Thus when abatement is too expensive, it is optimal to switch to a clean R&D policy, reconciling long-term growth with environmental concerns.

When the abatement technology is advanced enough, abatement becomes sufficiently cheap for cleaned dirty energy to be a viable alternative to renewable energy. When abatement takes place, the productivity index (A.15) can be written as

$$B^A = (A_c^{-\varphi} + A_{da}^{-\varphi})^{-\frac{1}{\varphi}},$$

where

$$A_{da} \stackrel{\text{def}}{=} [A_d^{-(1-a)} + A_a^{-(1-\alpha)}]^{-\frac{1}{1-\alpha}}.$$

With full abatement, final good production is given by the first line in (A.22). Since the aggregate machine cost is also proportional to B^A (cf. (A.18)) consumption will grow at the same rate as B^A. Maximizing the growth rate of consumption then amounts to maximizing the growth rate of B^A, which is given by

$$\dot{B}^A = \gamma\eta\{\chi s_c + (1-\chi)[(1-\zeta)s_d + \zeta s_a]\},$$

where

$$\chi \stackrel{\text{def}}{=} \frac{A_c^{-\varphi}}{A_c^{-\varphi} + A_{da}^{-\varphi}} \quad \text{and} \quad \zeta \stackrel{\text{def}}{=} \frac{A_a^{-(1-\alpha)}}{A_d^{-(1-\alpha)} + A_a^{-(1-\alpha)}}.$$

Logarithmic differentiation of χ gives

$$\dot{\chi} = (1-\dot{\chi})(-\varphi)[\gamma\eta s_c - \dot{A}_{da}(s_{da})],$$

where $s_{da} = 1 - s_c$ and $\dot{A}_{da}(s_{da})$ is the growth rate in efficiency parameter A_{da} when a fraction s_{da} of researchers are in an optimal way allocated to dirty energy and abatement technology.

Logarithmic differentiation of ζ gives

$$\dot{\zeta} = (1-\zeta)(1-\alpha)\gamma\eta[s_d - s_a].$$

The problem of maximizing the long-term growth in consumption is then

$$\max_{s_c, s_{da}} \gamma\eta\chi s_c + (1-\chi)\dot{A}_{da}(s_{da})$$
$$\text{s.t. } s_c + s_{da} = 1,$$

where

$$\dot{A}_{da}(s_{da}) = \max_{s_d, s_a} ((1-\zeta)s_d + \zeta s_a)\gamma\eta$$
$$\text{s.t. } s_d + s_a = s_{da}.$$

We first solve the second problem. If $s_d < \frac{s_{da}}{2} < s_a$, then $\dot{\zeta} < 0$, so that ζ converges to zero and $\dot{A}_{da}$ converges to $s_d\gamma\eta(< \frac{s_{da}}{2}\gamma\eta)$. On the other hand, if $s_d > \frac{s_{da}}{2} > s_a$ then $\dot{\zeta} > 0$, so that ζ will approach 1 and $\dot{A}_{da}$ converges to $s_a\gamma\eta(< \frac{s_{da}}{2}\gamma\eta)$. Finally, if $s_d = s_a = \frac{s_{da}}{2}$, then ζ remains constant and $\dot{A}_{da} = \frac{s_{da}}{2}\gamma\eta$. Therefore, the optimal R&D

strategy is to allocate the fraction s_{da} evenly across the dirty energy sector and the abatement sector: $\dot{A}_{da}(s_{da}) = \frac{s_{da}}{2}\gamma\eta$.

Now we can solve the former problem

$$\max_{0<s_c<1} \dot{B}^{FA} = \gamma\eta\chi s_c + (1-\chi)\frac{1-s_c}{2}\gamma\eta = \gamma\eta\left(\frac{3\chi-1}{2}s_c + \frac{1-\chi}{2}\right).$$

The growth in χ can now be written as

$$\begin{aligned}\dot{\chi} &= (1-\chi)(-\varphi)\left[\gamma\eta s_c - \frac{1-s_c}{2}\gamma\eta\right],\\ &= -\varphi(1-\chi)\gamma\eta\left(\frac{3}{2}s_c - \frac{1}{2}\right).\end{aligned}$$

Since $\varepsilon > 1$, $\varphi < 0$. If $s_c > \frac{1}{3}$, then $\dot{\chi} > 0$ so that χ will converge to 1 and $\dot{B}^{\mathrm{FA}}$ converges to $\gamma\eta s_c$. If $s_c < \frac{1}{3}$, then $\dot{\chi} < 0$ so that χ will converge to 0 and $\dot{B}^{\mathrm{FA}}$ to $\frac{1-s_c}{2}\gamma\eta$. Finally, if $s_c = \frac{1}{3}$ then χ remains constant and $\dot{B}^{\mathrm{FA}}$ converges to $\frac{1}{3}\gamma\eta$. Therefore, consumption growth is a convex function of s_c (∨-shaped), with two local maxima: (i) $s_c = 0$ and $s_d = s_a = \frac{1}{2}$ giving a LT growth rate of $\frac{\gamma\eta}{2}$ and (ii) $s_c = 1$ and $s_d = s_a = 0$ giving a LT growth rate of $\gamma\eta$. The faster growth rate in the last case goes at the cost of a higher LT temperature increase. So there is a trade-off. For $\varepsilon = 3$ and $\varphi = -\frac{4}{3}$, both policies are about equally good when $\mathrm{MC}_a = 0.30$, as in Fig. 4.

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